

Universal Model

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$\neg A$ means that A is refutable



Refutation System

$$\frac{}{\neg p} \quad \frac{\neg \sigma A}{\neg A} \quad \frac{A \vdash B \quad \neg B}{\neg A}$$



Refutation System

$$\frac{}{\neg p} \quad \frac{\neg \sigma A}{\neg A} \quad \frac{A \vdash B \quad \neg B}{\neg A}$$

Łukasiewicz (1951)

$A \in \mathcal{L}(X)$ classically derivable iff
 $v \models A$ for all $v : X \rightarrow \{0, 1\}$.



Intuitionistic Propositional Logic



Intuitionistic Propositional Logic

IPC



Łukasiewicz (1952)

$$\frac{}{\neg p} \quad \frac{\neg \sigma A}{\neg A} \quad \frac{A \vdash B \quad \neg B}{\neg A}$$

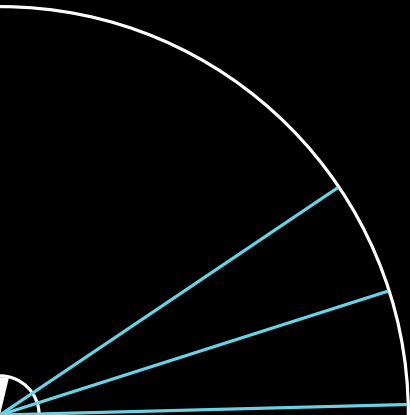
$$\frac{\neg A \quad \neg B}{\neg A \vee B}$$



Overview



Overview

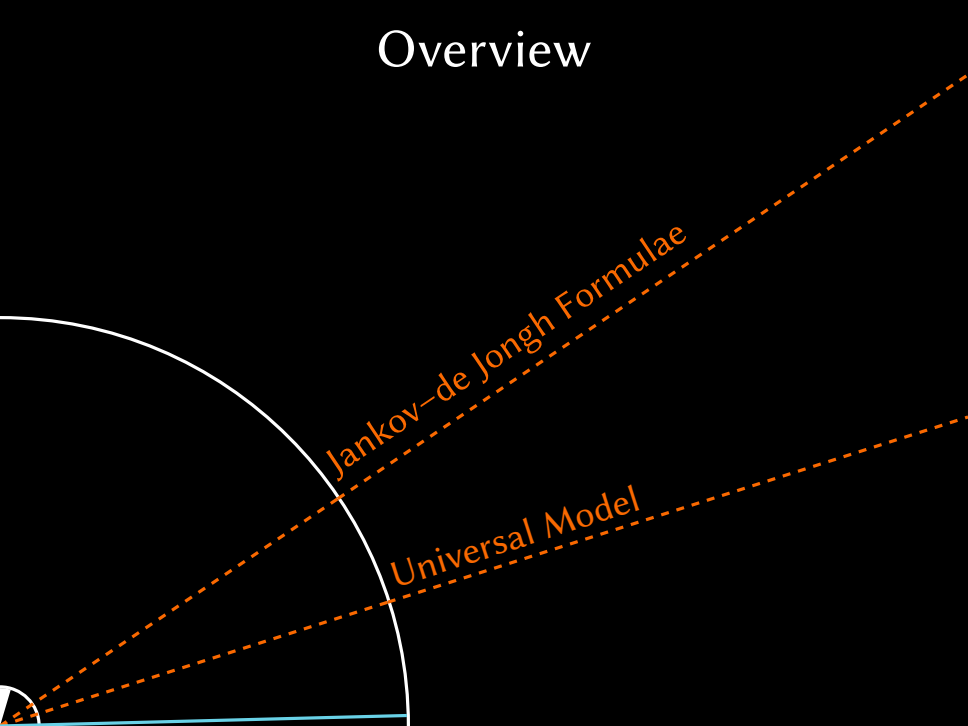


Overview

Jankov-de Jongh Formulae



Overview



Overview

Jankov-de Jongh Formulae

Universal Model

Syntactic Characterisation of IPC

Kripke models



Kripke models

reflexive



Kripke models

reflexive · transitive



Kripke models

reflexive · transitive · anti-symmetric



Kripke models

reflexive · transitive · anti-symmetric
preservation of truth



Image-finite



Image-finite

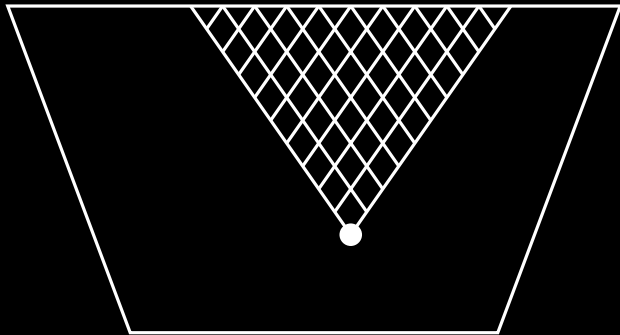
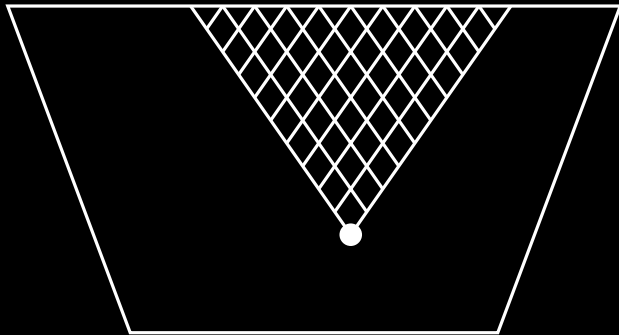
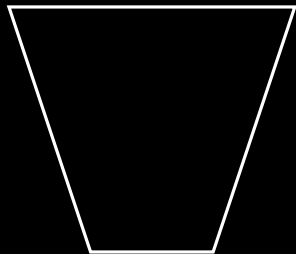
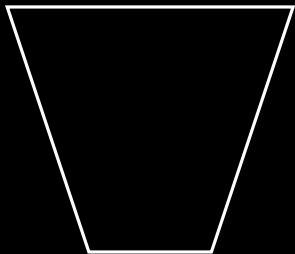


Image-finite

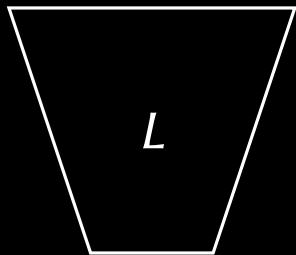
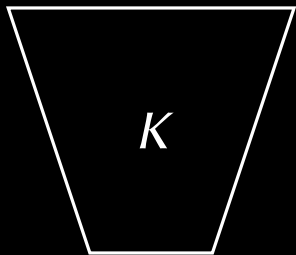
finite



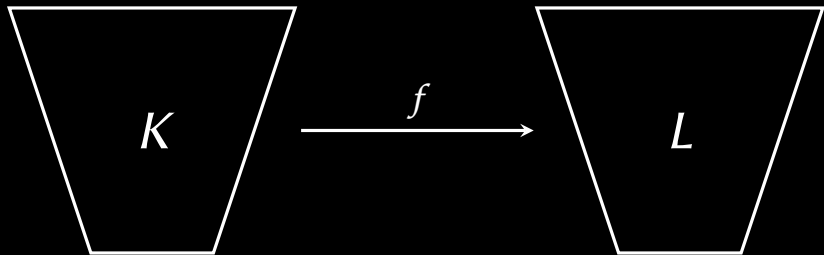
Morphisms



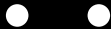
Morphisms



Morphisms



Analogous



Analogous

The image features a black background with the word "Analogous" centered at the top in a white serif font. Below the text, two white dots are positioned side-by-side at the bottom. From each dot, two white lines extend upwards and outwards, crossing each other in the center to form two overlapping V-shapes that point towards the top of the frame.

Analogous

The image features a black background with white lines and text. At the top, the word "Analogous" is written in a white, serif font. Below the text, two V-shaped structures are drawn with white lines. Each V-shape is formed by two lines that meet at a central point. The two V-shapes are positioned symmetrically, with their central points facing each other. Below the meeting point of the two V-shapes, there are two small, solid white circles arranged horizontally. In the bottom-left corner, there is a small, white, semi-circular logo consisting of several radiating lines.


Analogous

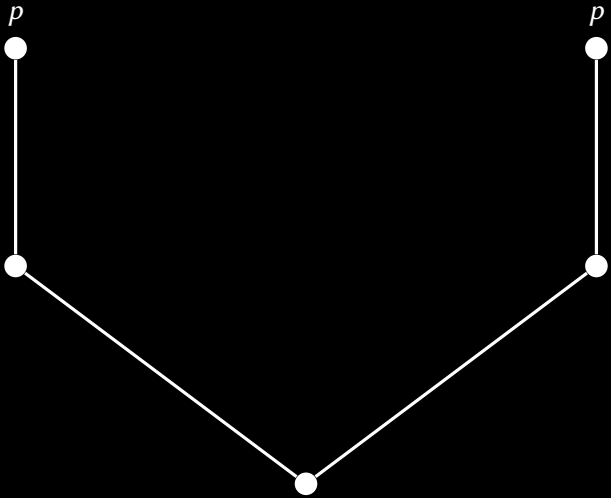
A diagram consisting of a large inverted V-shape formed by two white lines extending from the top corners towards the center. At the bottom vertex of the V, there is a small, upward-curving white arc. Below this arc, there are two small white dots positioned horizontally next to each other.

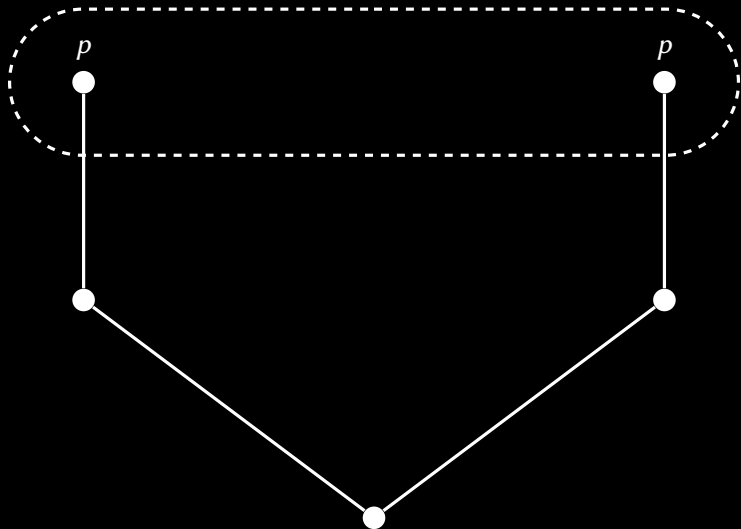
Analogous

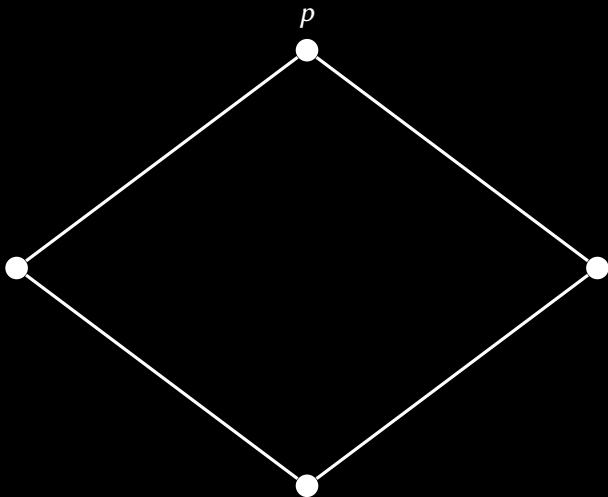


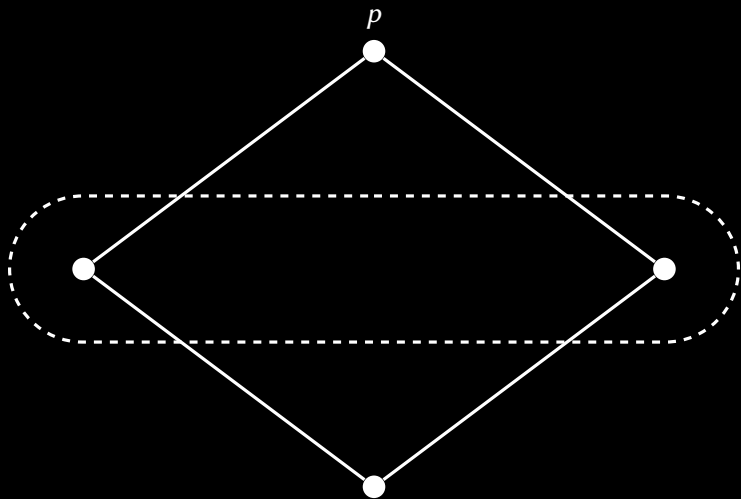
$k \equiv l$ when $v(k) = v(l)$ and $k \leq u$ iff $l \leq u$ for all $u \neq k, l$





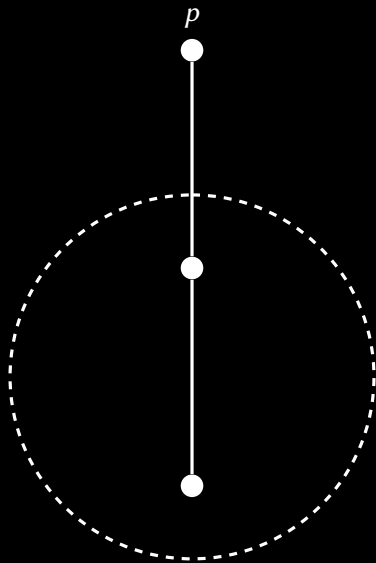






p





p



A model is **concrete** when
analogous nodes are **equal**.



In concrete & image-finite models,
upsets & complements of downsets
are **definable**.



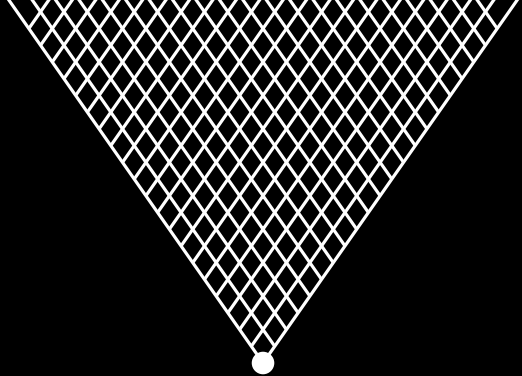
In concrete & image-finite models,
upsets & complements of downsets
are **definable**.

de Jongh (1968)



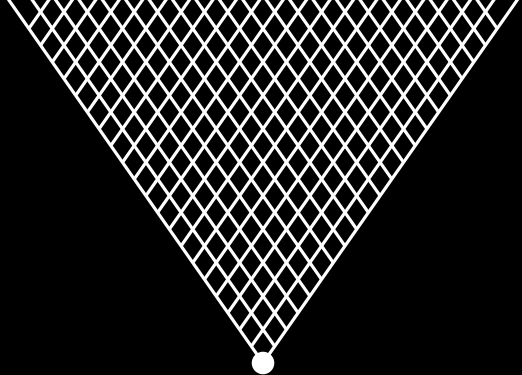


•
k



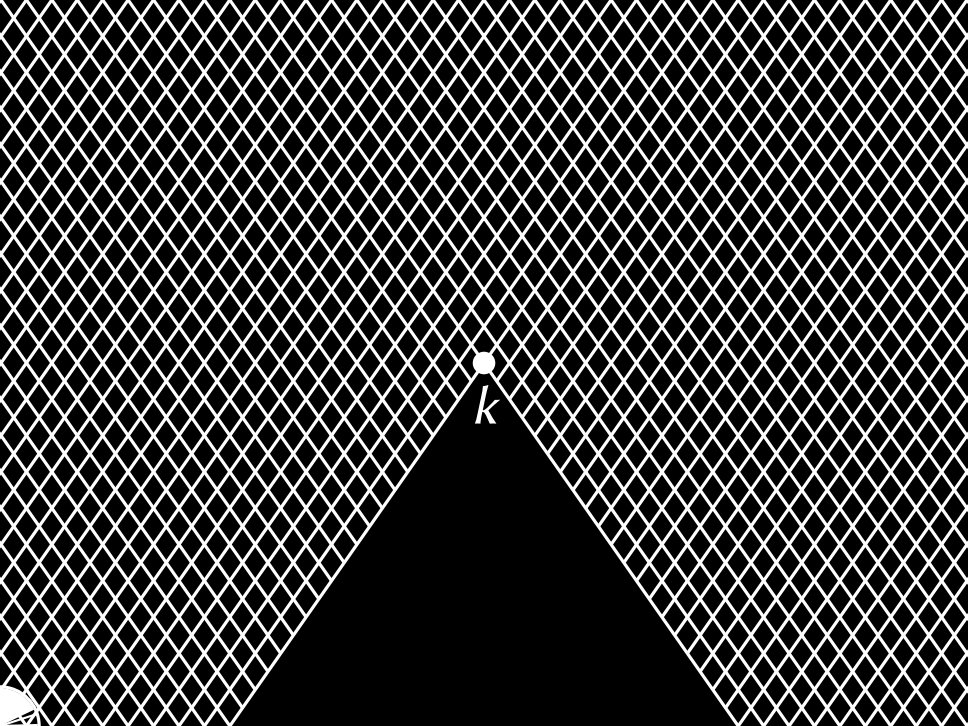
k



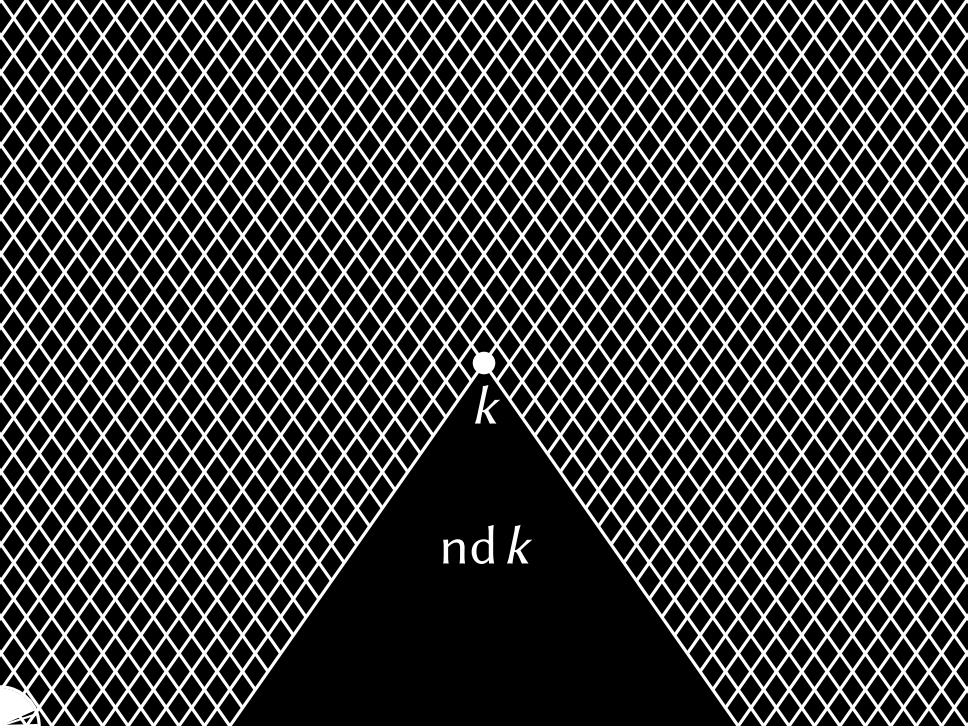


k

up k



k



k

$nd k$

$$\text{up } k := \bigwedge_{k \Vdash p} p \wedge \left(\begin{array}{l} \bigvee_{W \Vdash p} p \vee \bigvee_{w \in W} \text{nd } w \rightarrow \bigvee_{w \in W} \text{up } w \\ k \nVdash p \end{array} \right)$$

$$\text{up } k := \bigwedge_{k \Vdash p} p \wedge \left(\begin{array}{l} \bigvee_{W \Vdash p} p \vee \bigvee_{w \in W} \text{nd } w \rightarrow \bigvee_{w \in W} \text{up } w \\ k \nVdash p \end{array} \right)$$

$$\text{nd } k := \text{up } k \rightarrow \bigvee_{w \in W} \text{up } w$$

A model is **universal**
when it is
the **terminal image-finite** model.

A model is **universal**
over X when it is
the **terminal image-finite** model
over X .

The universal model is complete.

The universal model is **complete**.

Bellissima (1986) · Rybakov (1994)

The universal model is **complete**.

Bellissima (1986) · Rybakov (1994)

Shehtman (1978) · Grigolia (1987)

$$\text{Univ } X \cong \text{colim} \left(\begin{array}{c} \text{FRKripke } X \\ \cap \\ \text{Kripke } X \end{array} \right)$$

The universal model is **concrete**.

The **image-finite part** of the **canonical** model is the universal model.

$$A \rightarrow nd k$$

$A \rightarrow \text{nd } k$

for all $l \Vdash A$ have $l \not\leq k$

$A \rightarrow \text{nd } k$

for all $l \Vdash A$ have $l \not\leq k$

$k \not\Vdash A$

Skura (1989)

$$\frac{}{\neg p} \quad \frac{\neg \sigma A}{\neg A} \quad \frac{A \vdash B \quad \neg B}{\neg A}$$

$$\frac{\neg (\bigvee B_i \rightarrow A) \rightarrow B_j \text{ for all } j}{\neg (\bigvee B_i \rightarrow A) \rightarrow \bigvee B_i}$$

IPC is the intermediate logic where

$$\not\vdash = \neg$$



References I



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References II



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