



The Admissible Rules of Subframe Logics

Jeroen Goudsmit

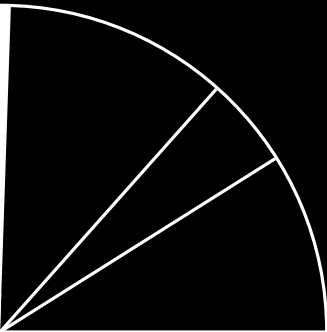
Utrecht University

July 19th 2014, 15:00 – 15:30

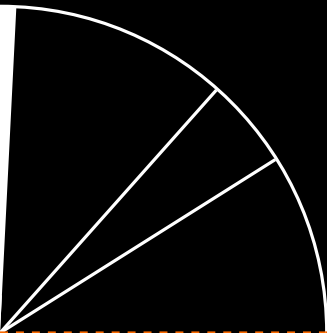
Overview



Overview



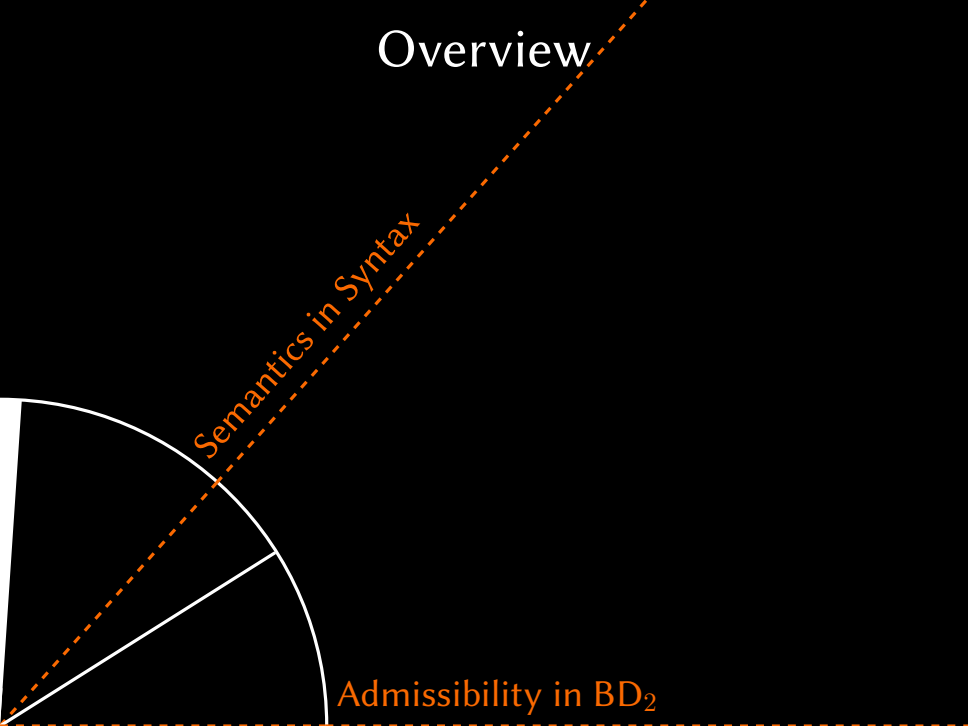
Overview



Admissibility in BD_2

Overview

Semantics in Syntax



Admissibility in BD_2

Overview

Semantics in Syntax

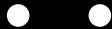
Weak Visser Rules

Admissibility in BD_2

Semantics in Syntax



Analogous



Analogous

The image features a black background with the word "Analogous" centered at the top in a white serif font. Below the text, two small white circular dots are positioned horizontally. From each dot, two white lines extend upwards and outwards, crossing each other in the center to form two overlapping V-shapes. The lines from the left dot extend towards the top-left and top-right, while the lines from the right dot extend towards the top-right and top-left. In the bottom-left corner, there is a small, partially visible white circular icon with several radial lines, resembling a fan or a stylized wheel.

Analogous

The diagram consists of two V-shaped structures, one on the left and one on the right, both pointing downwards. The two inner lines of each V meet at a central point. Below this central point, there are two small white dots positioned horizontally next to each other. The entire diagram is rendered in white lines and dots on a black background.


Analogous

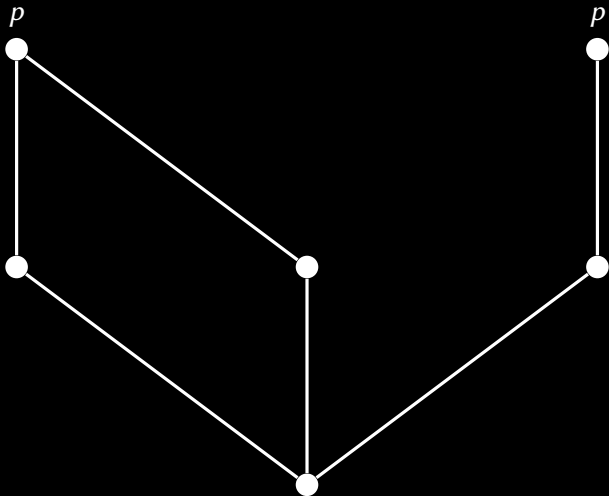
A diagram consisting of a large inverted V-shape formed by two white lines extending from the top corners towards the center. At the bottom vertex of the V, there is a small, upward-curving white arc. Below this arc, there are two small white dots positioned horizontally next to each other.

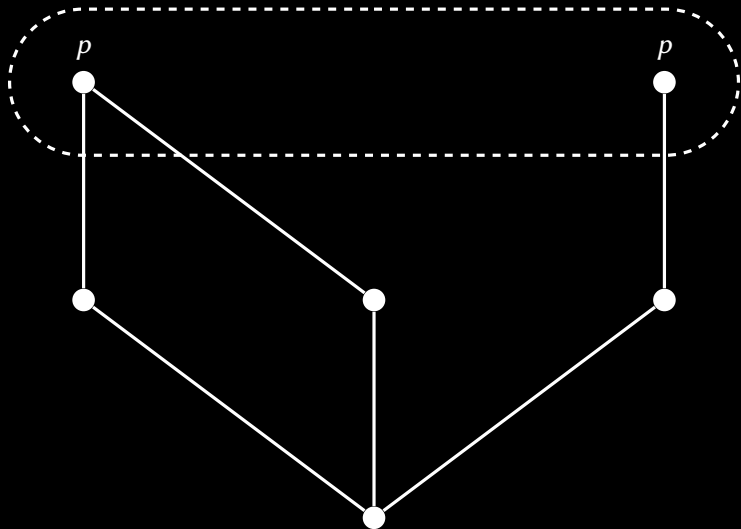
Analogous

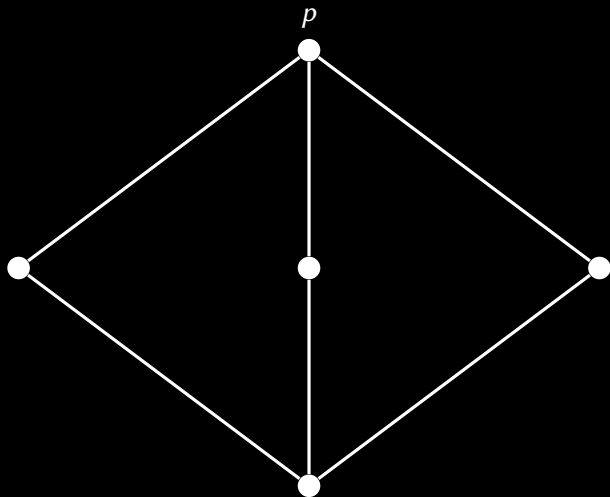


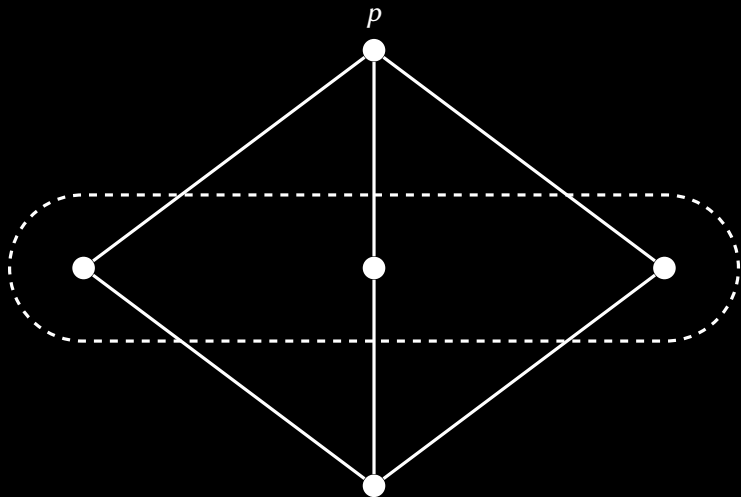
$k \equiv l$ when $v(k) = v(l)$ and $k \leq u$ iff $l \leq u$ for all $u \neq k, l$





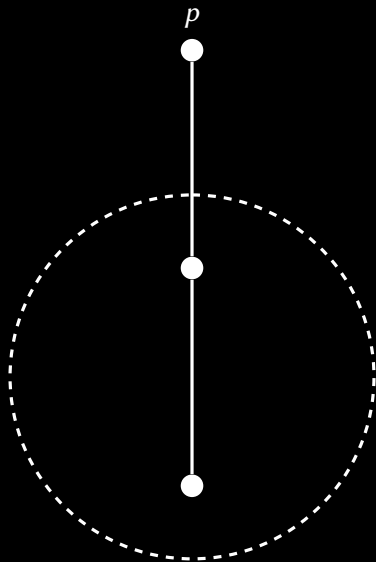






p





p



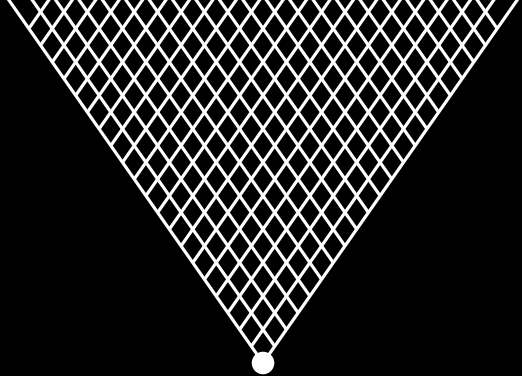
Call a model *concrete* whenever
analogous equals equal.





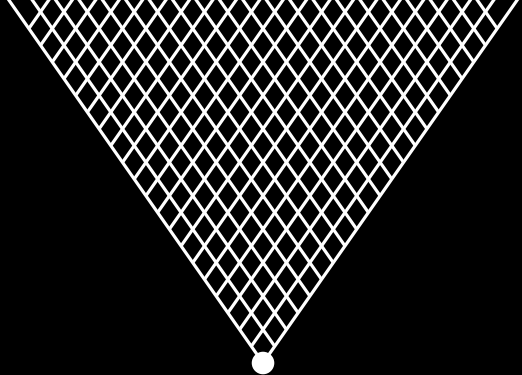
•
k





k

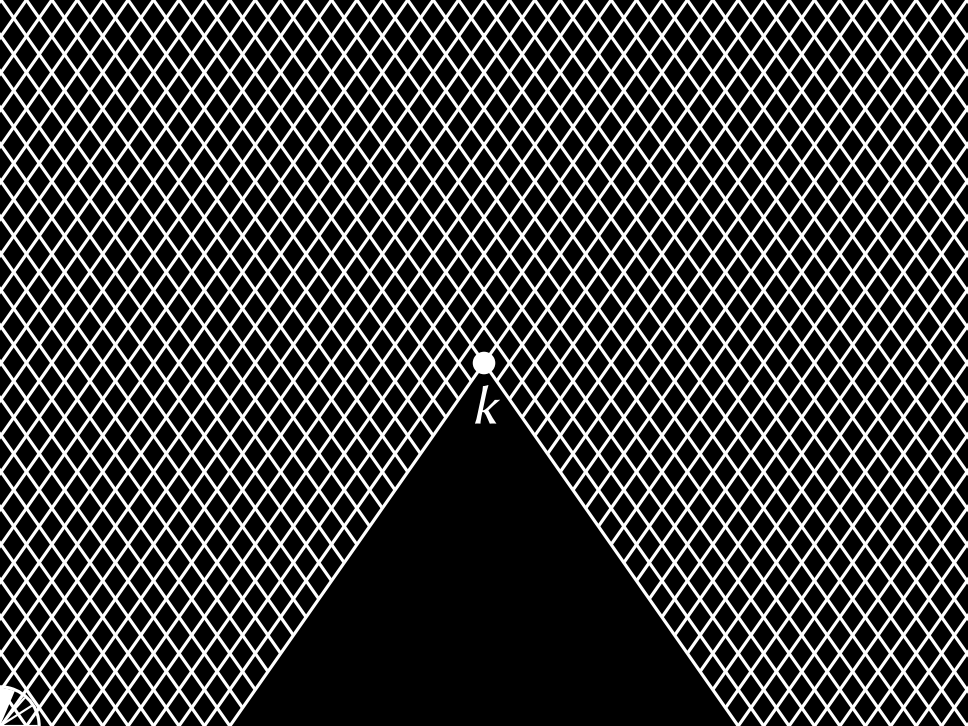




k

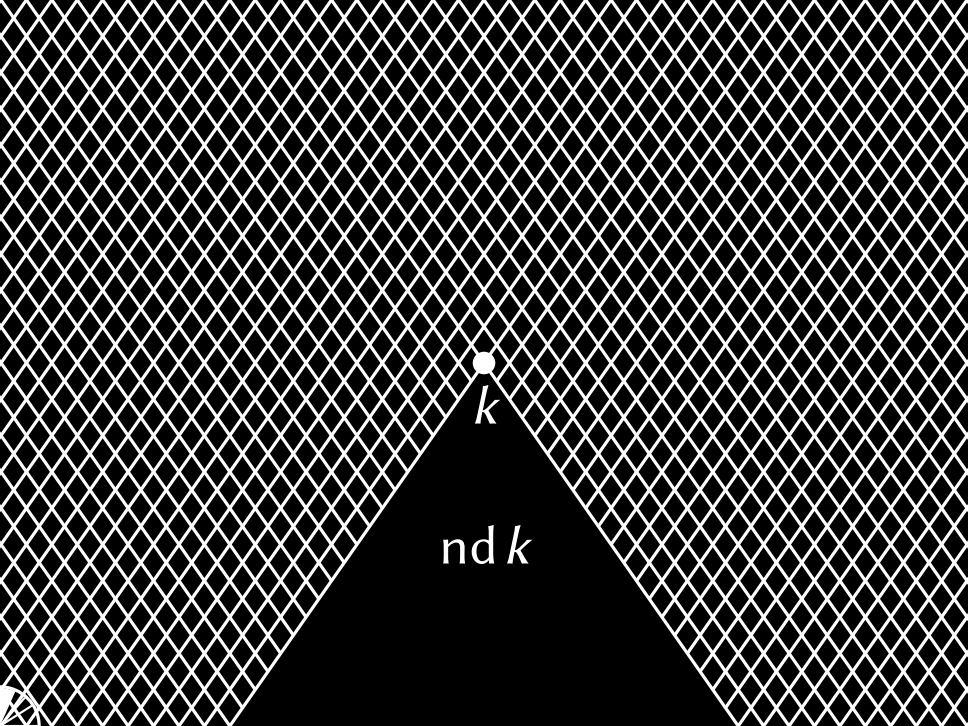
up k





k

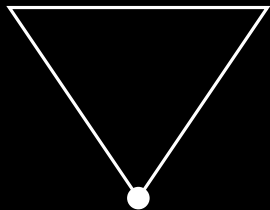
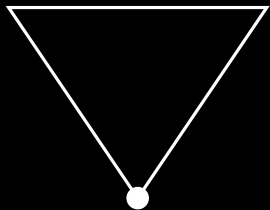
k

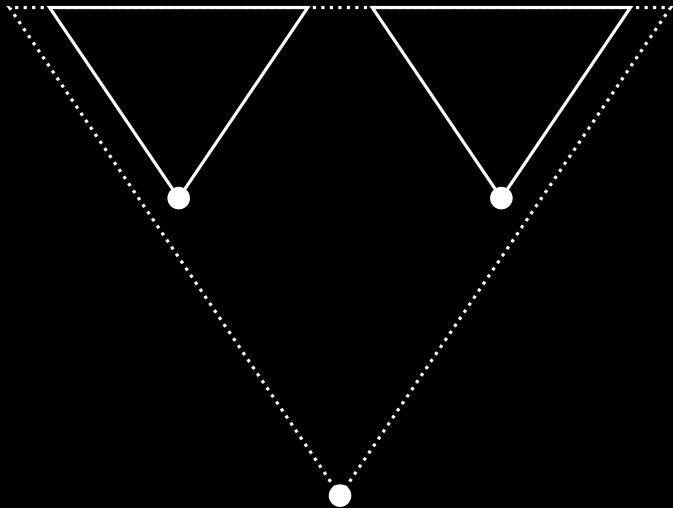


k

$nd k$







$$K \Vdash A \vee B$$

$$K \Vdash A \text{ or } K \Vdash B$$


syntax

$$K \Vdash A \vee B$$

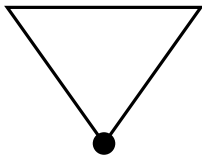
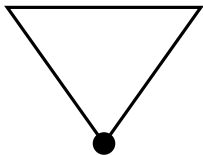
$$K \Vdash A \text{ or } K \Vdash B$$


semantics

syntax

$$K \Vdash A \vee B$$

$$K \Vdash A \text{ or } K \Vdash B$$

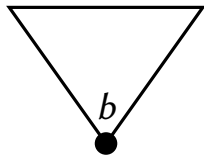
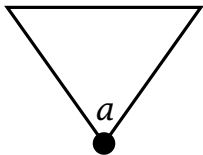



semantics

syntax

$$K \Vdash A \vee B$$

$$K \Vdash A \text{ or } K \Vdash B$$

semantics

syntax

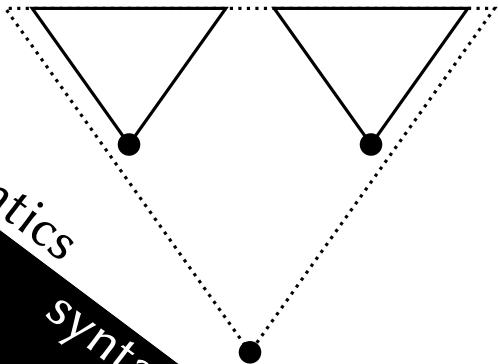
$K \Vdash A \vee B$

$K \Vdash A$ or $K \Vdash B$



semantics

syntax

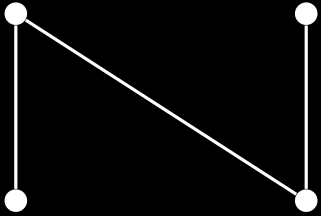


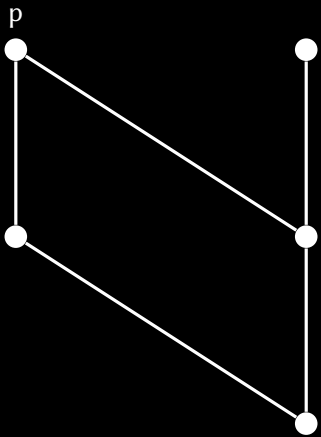
$K \Vdash A \vee B$

$K \Vdash A \text{ or } K \Vdash B$

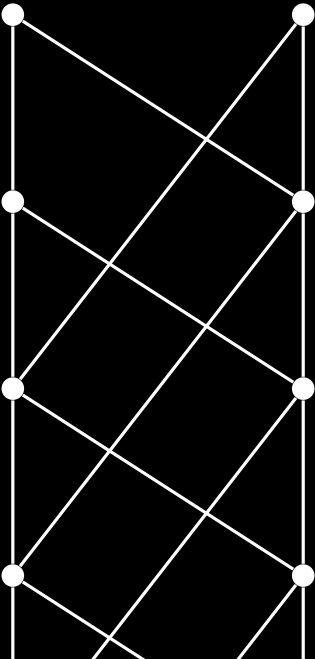


p





p



There is a concrete & complete
model to each finite set of
variables.



There is a concrete & complete
model to each finite set of
variables X ;
the universal model $U(X)$.





1957 Rieger

1960 Nishimura

1968 de Jongh

1973 Urquhart

1975 Esakia and Grigolia

1978 Shehtman

1986 Bellissima

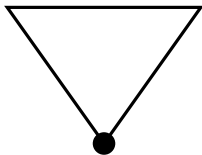
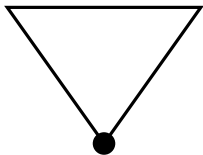


semantics

syntax

$A \vee B$

A, B



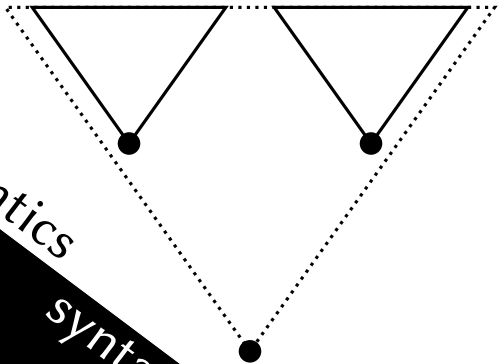
semantics

syntax

$$A \vee B$$
$$A, B$$

semantics

syntax



$$A \vee B$$
$$A, B$$


A / Δ admissible



if σA is derivable



A / Δ admissible



then σC is derivable, for some $C \in \Delta$



$A \vee B$

 A, B

$$\neg C \rightarrow A \vee B$$

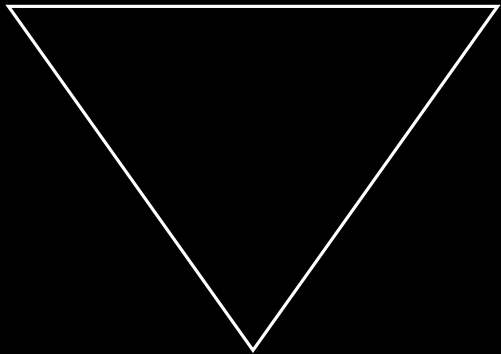
$$(\neg C \rightarrow A) \vee (\neg C \rightarrow B)$$

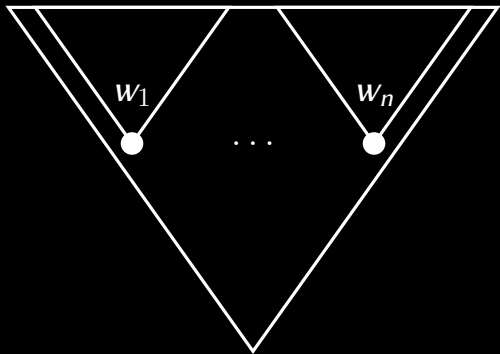
Weak Visser Rules

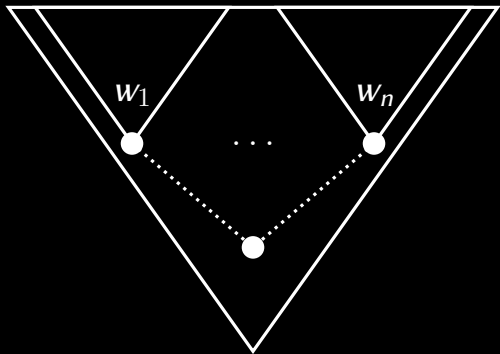


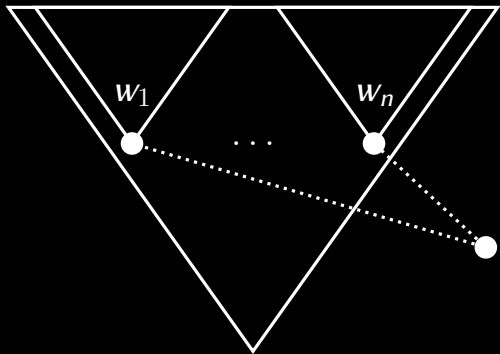
$$(\bigvee \Delta \rightarrow A) \rightarrow \bigvee \Delta$$

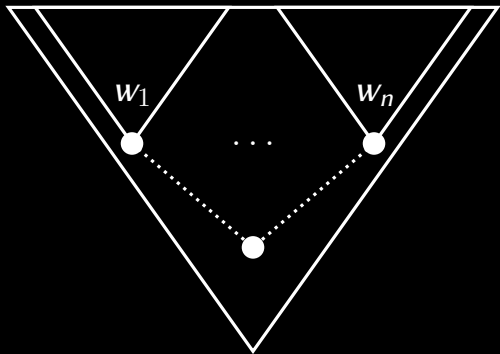
$$\bigvee \{(\bigvee \Delta \rightarrow A) \rightarrow C\}_{C \in \Delta}$$



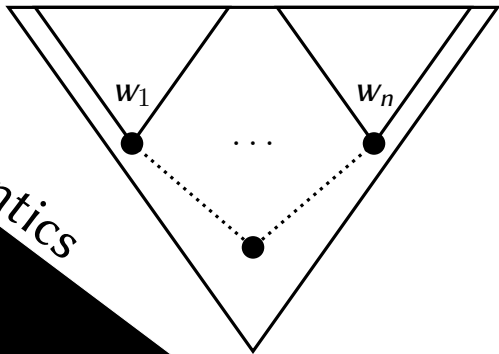






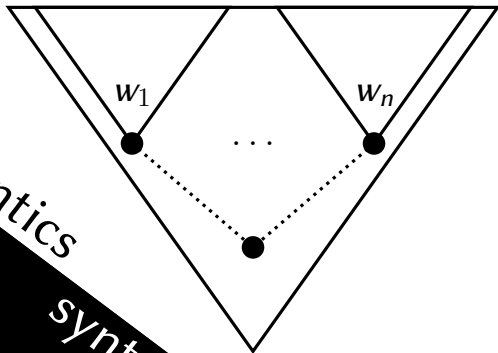


semantics

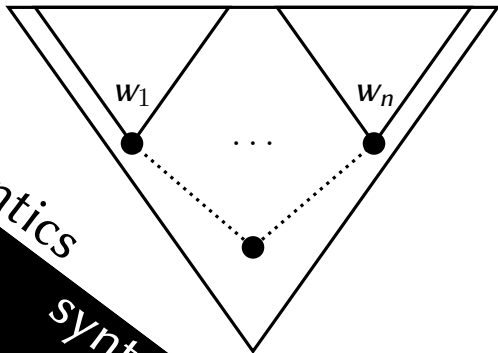


semantics

syntax



semantics

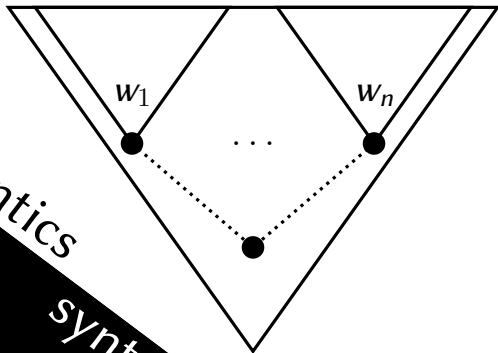


syntax

$$\left(\bigvee_{i=1}^n \text{nd } w_i \rightarrow \bigvee_{i=1}^n \text{up } w_i \right) \rightarrow \bigvee_{i=1}^n \text{nd } w_i$$

$$\bigvee_{j=1}^n \left(\bigvee_{i=1}^n \text{nd } w_i \rightarrow \bigvee_{i=1}^n \text{up } w_i \right) \rightarrow \text{nd } w_j$$

semantics



syntax

$$\left(V \Delta \rightarrow A \right) \rightarrow V \Delta$$

$$\bigvee_{C \in \Delta} \left(V \Delta \rightarrow A \right) \rightarrow C$$

A model K is *closed under* A/Δ
whenever
 $K \Vdash A$ yields $K \Vdash C$ for some $C \in \Delta$.

Theorem (G, 2013):

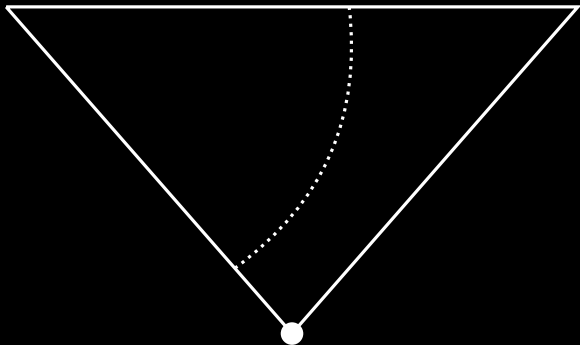
Any concrete model satisfies the weak extension property iff it is closed under the weak Visser rules.

Corollary:

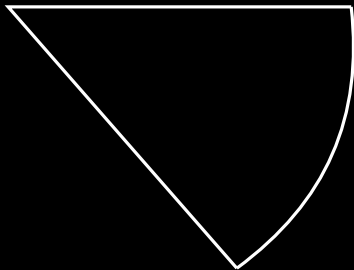
An intermediate logic with the finite model property has the weak extension property iff it admits the weak Visser rules.

Subframe logics are the logics that are closed under taking subframes.

Subframe logics are the logics that are closed under taking subframes.



Subframe logics are the logics that are closed under taking subframes.



Theorem:
Subframe logics enjoy the
weak extension property.

Theorem (Skura, 1989):
IPC is the only intermediate logic
with the weak Visser rules and
disjunction property.

Admissibility in BD_2

Logic of Depth n

$$\mathbf{bd}_0 = \perp$$

$$\mathbf{bd}_{n+1} = p_{n+1} \vee (p_{n+1} \rightarrow \mathbf{bd}_n).$$

$$\text{BD}_n := \text{IPC} + \text{bd}_n$$

BD_2 is the logic of frames of height at most two.

Does BD_2 admit the
Weak Visser rules?

$$(\bigvee \Delta \rightarrow A) \rightarrow \bigvee \Delta$$

$$\bigvee \{(\bigvee \Delta \rightarrow A) \rightarrow C\}_{C \in \Delta}$$

BD_2 *does* admit the
Weak Visser rules.

$$(\bigvee \Delta \rightarrow A) \rightarrow \bigvee \Delta$$

$$\bigvee \{(\bigvee \Delta \rightarrow A) \rightarrow C\}_{C \in \Delta}$$

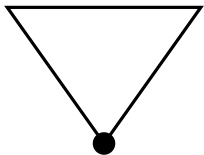
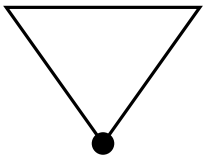
Does BD_2 enjoy
the disjunction property?

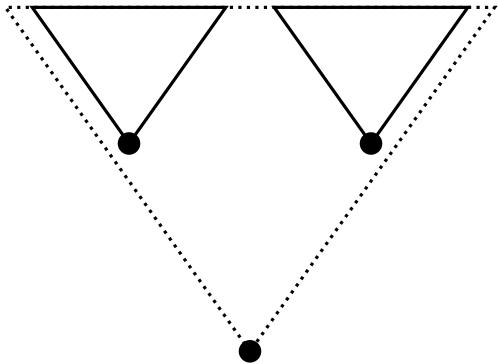
$$\frac{A \vee B}{A, B}$$

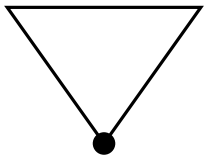
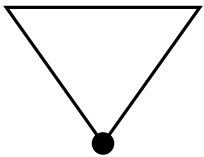
BD_2 *does not* enjoy
the disjunction property.

$$\frac{A \vee B}{A, B}$$

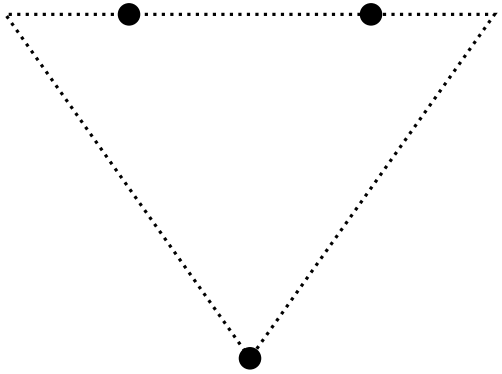


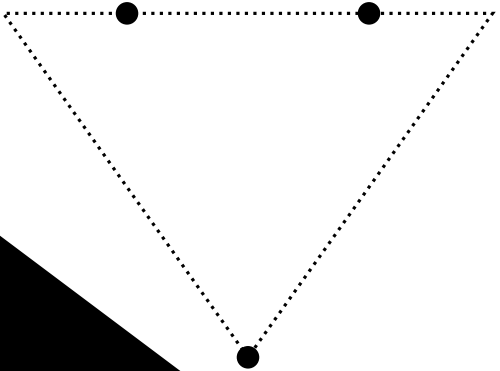
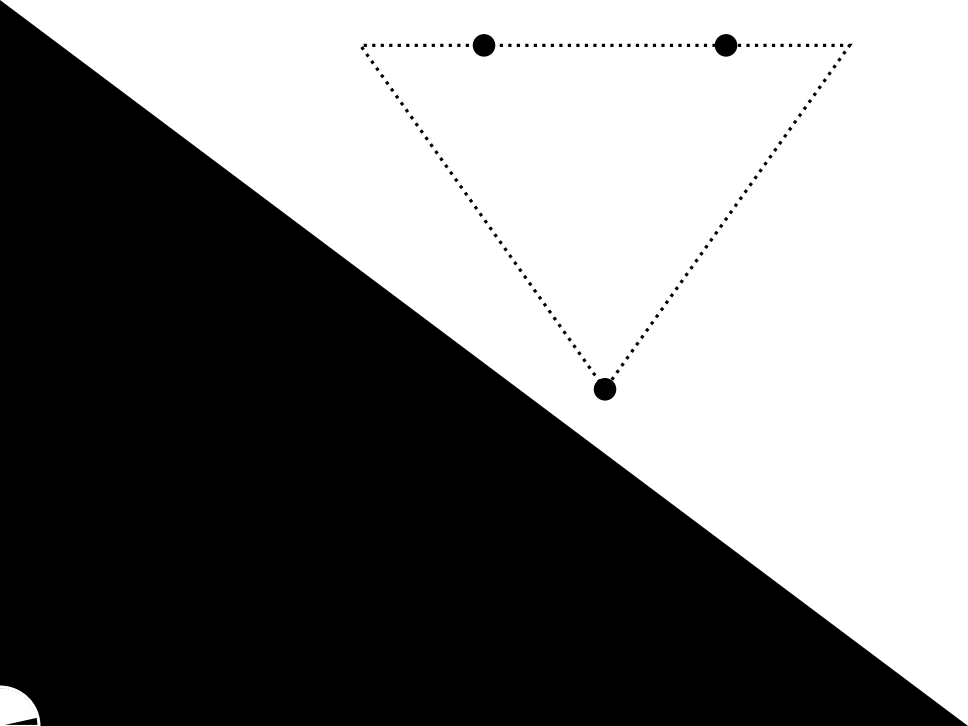




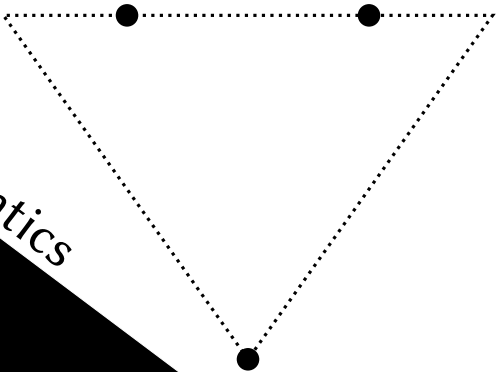






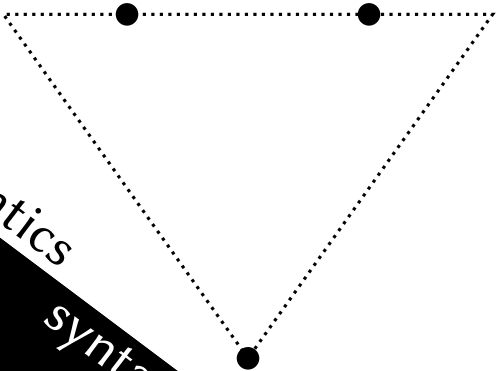


semantics



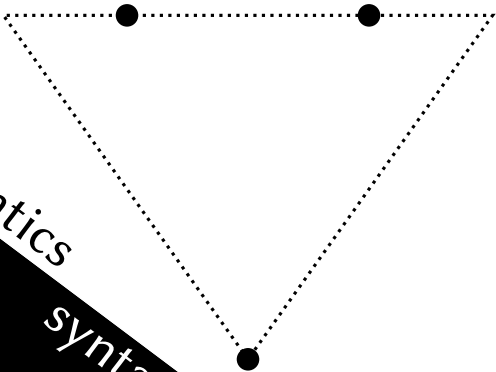
semantics

syntax



semantics

syntax


$$A \vee B$$

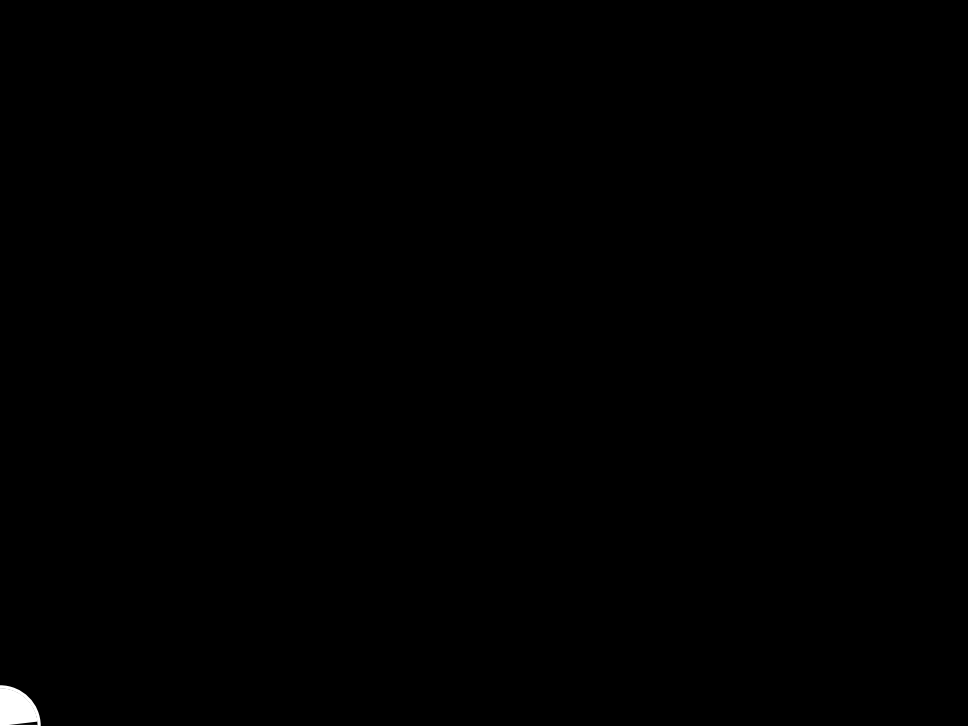
$$\neg\neg A, \neg\neg B$$

Does BD_2 enjoy the doubly-negated disjunction property?

$$\frac{A \vee B}{\neg\neg A, \neg\neg B}$$

BD_2 *does* enjoy the doubly-negated disjunction property.

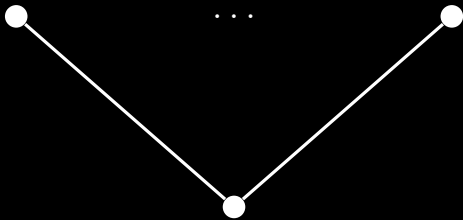
$$\frac{A \vee B}{\neg\neg A, \neg\neg B}$$



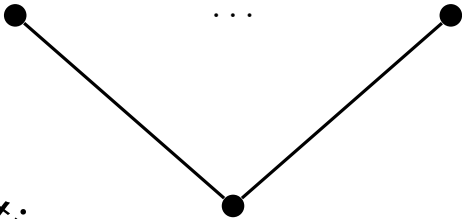


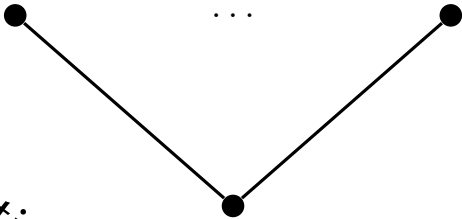
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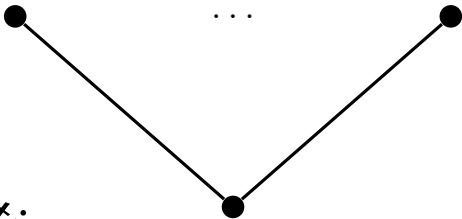
semantics





semantics

syntax



semantics

syntax

$$(\forall \Delta \rightarrow A) \rightarrow \forall \Delta$$

$$\{ \neg \neg (\forall \Delta \rightarrow A) \rightarrow C \}_{C \in \Delta}$$

Theorem (G 2013):






A concrete model is closed under all admissible rules of BD_2 iff it is closed under:

$$\frac{(\bigvee \Delta \rightarrow A) \rightarrow \bigvee \Delta}{\{\neg\neg(\bigvee \Delta \rightarrow A) \rightarrow C\}_{C \in \Delta}}$$

Theorem (G 2013):
All admissible rules of BD_2
follow from:

$$\frac{(\bigvee \Delta \rightarrow A) \rightarrow \bigvee \Delta}{\{\neg\neg(\bigvee \Delta \rightarrow A) \rightarrow C\}_{C \in \Delta}}$$



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-  **de Jongh, D. H. J. (1968).** “Investigations on the Intuitionistic Propositional Calculus”. PhD thesis. University of Wisconsin.
-  **Esakia, L. and R. Grigolia (1975).** “Christmas trees. On free cyclic algebras in some varieties of closure algebras”. In: *Bulletin of the Section of Logic* 4.3. ISSN: 0138-0680.
-  **Goudsmit, J. P. (2013).** “The Admissible Rules of BD_2 and GSc ”. In: *Logic Group Preprint Series* 313. To appear in the Notre Dame Journal of Formal Logic, pp. 1–24. URL: <http://phil.uu.nl/preprints/lgps/number/313>.
-  **Nishimura, I. (1960).** “On Formulas of One Variable in Intuitionistic Propositional Calculus”. In: *The Journal of Symbolic Logic* 25.4, pp. 327–331. ISSN: 00224812. DOI: [10.2307/2963526](https://doi.org/10.2307/2963526).



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Skura, T. F. (1989). “A complete syntactical characterization of the intuitionistic logic”. In: *Reports on Mathematical Logic* 23, pp. 75–80.



Urquhart, A. (1973). “Free Heyting algebras”. In: *Algebra Universalis* 3.1, pp. 94–97. ISSN: 0002-5240. DOI: [10.1007/BF02945107](https://doi.org/10.1007/BF02945107).