The Admissible Rules of Subframe Logics

Jeroen Goudsmit
Utrecht University
July 19^{th} 2014, 15:00 – 15:30

Overview

Overview

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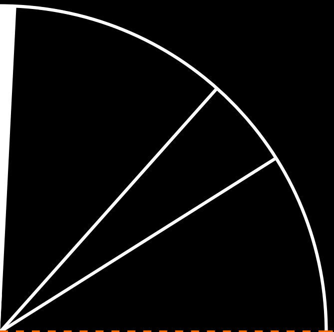
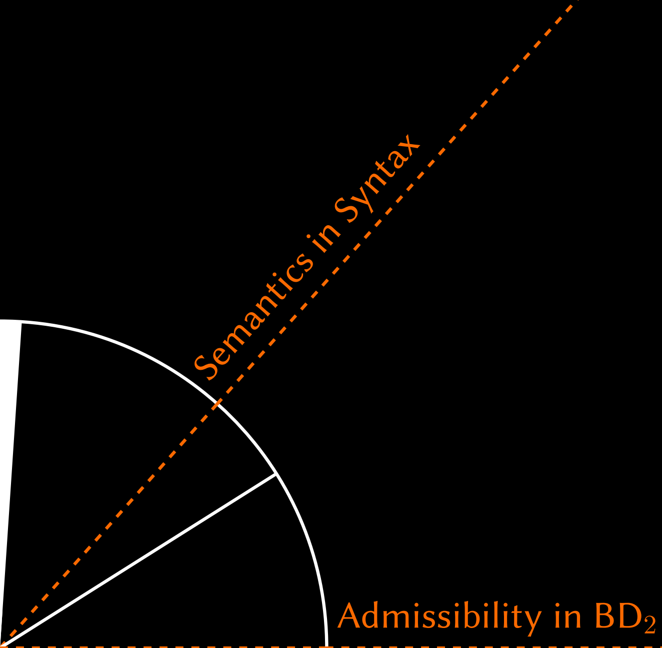
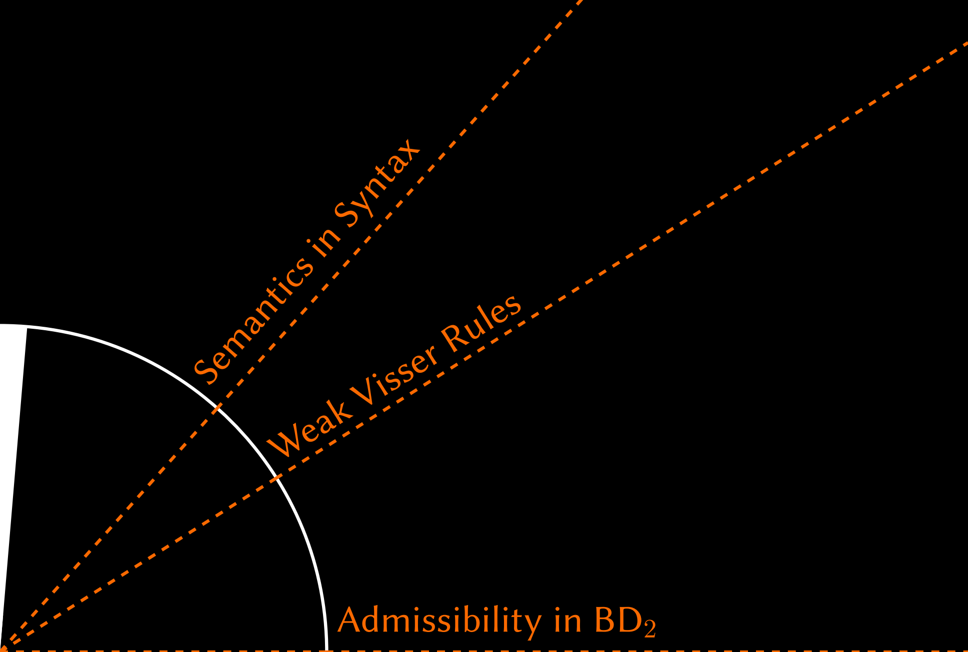


Admissibility in BD_2

Overview



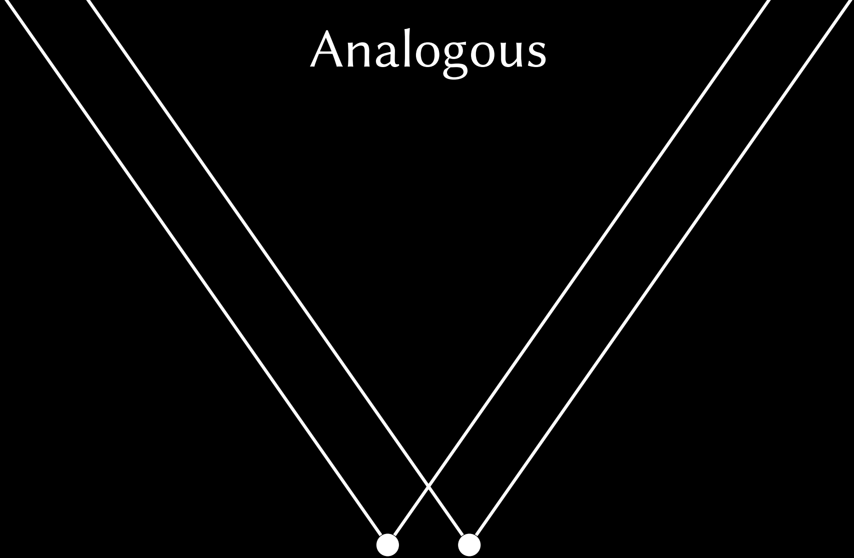
Overview


Semantics in Syntax
Weak Visser Rules
Admissibility in BD_2

Semantics in Syntax

Analogous

Analogous



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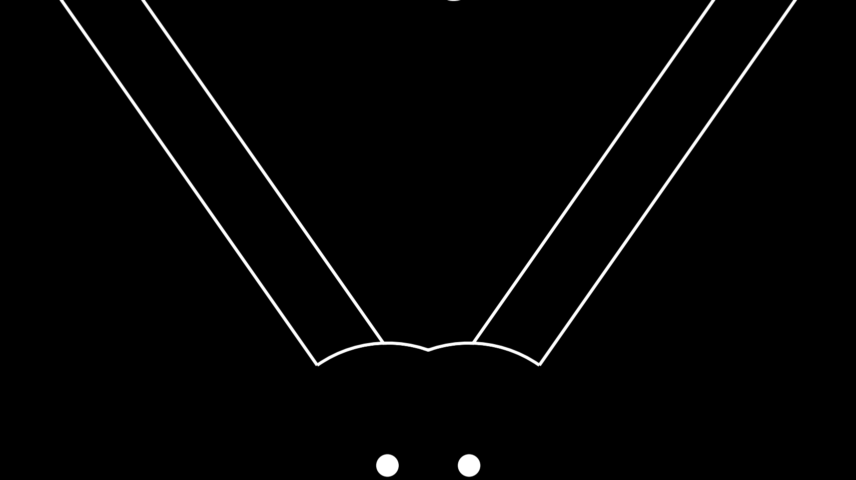

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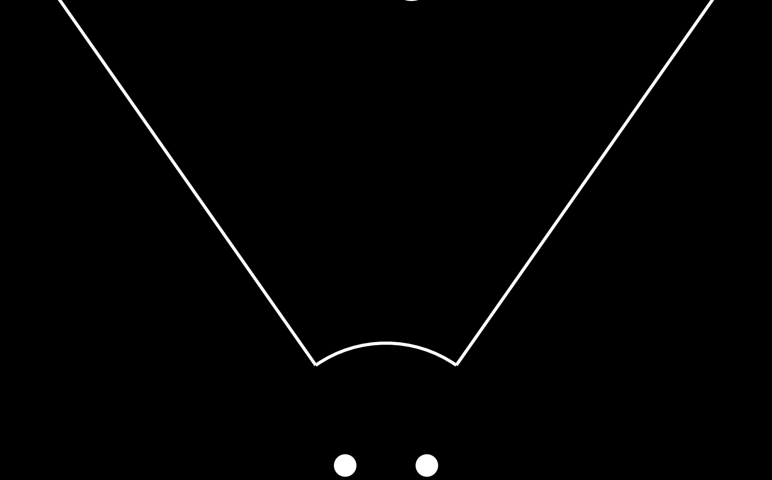

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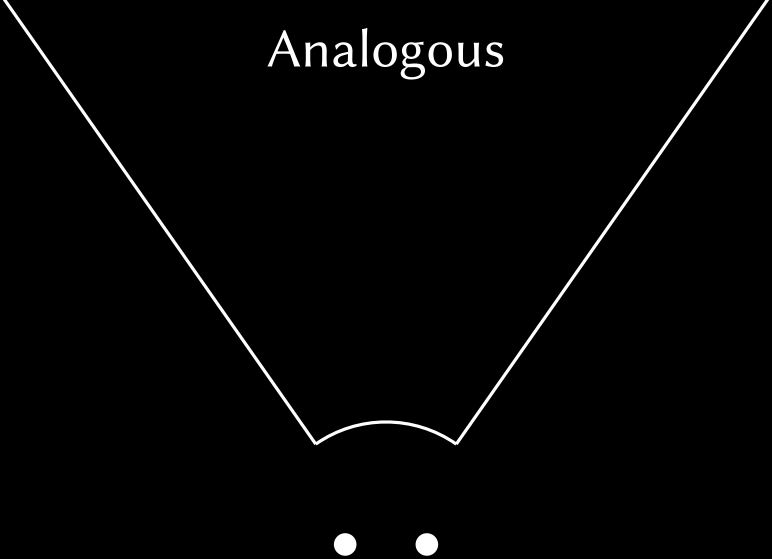
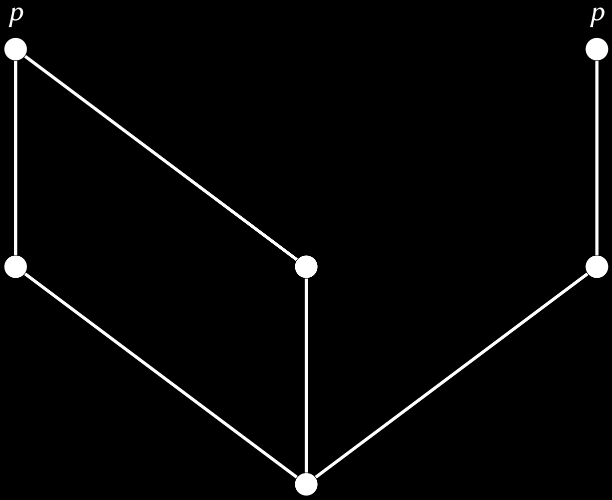


$k \equiv l$ when $v(k) = v(l)$ and $k \leq u$ iff $l \leq u$ for all $u \neq k, l$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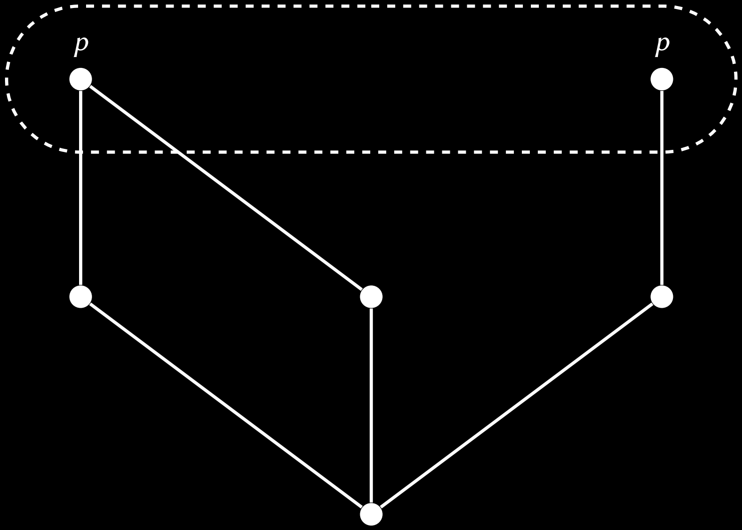
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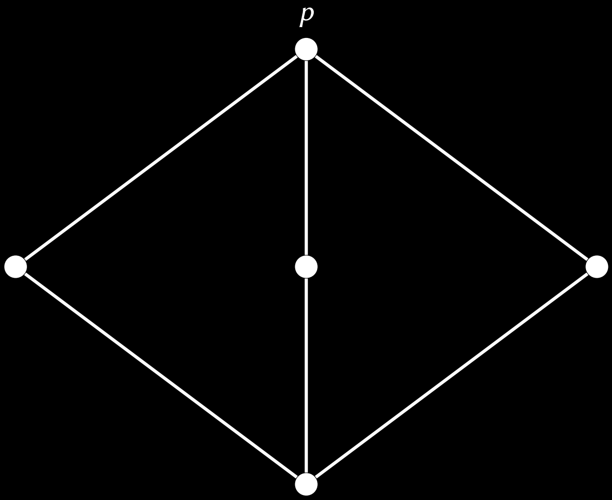
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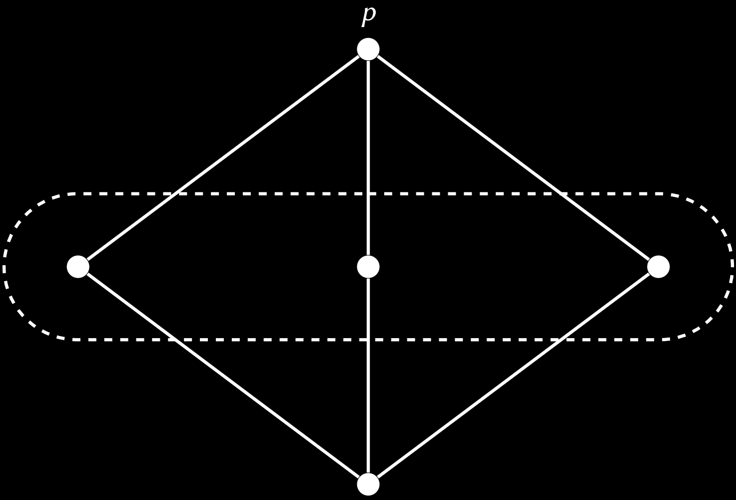
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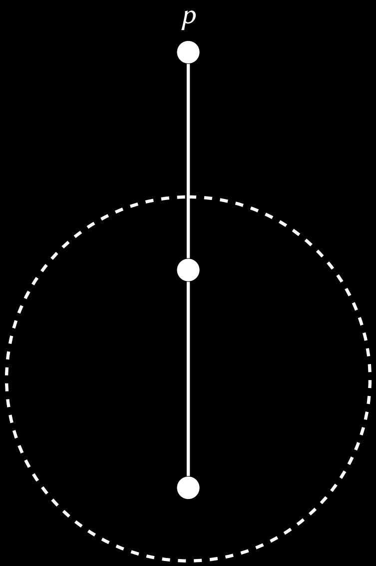
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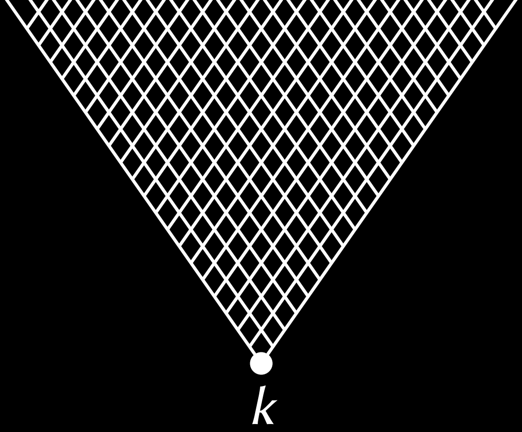
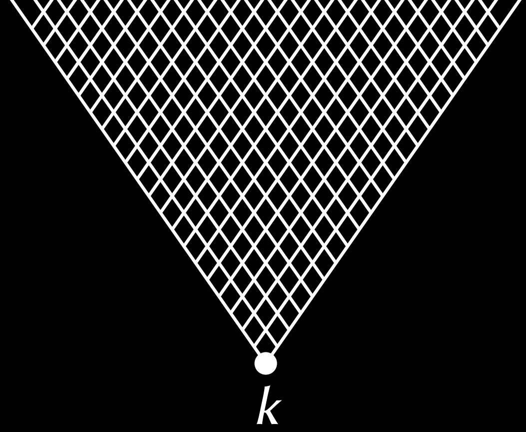
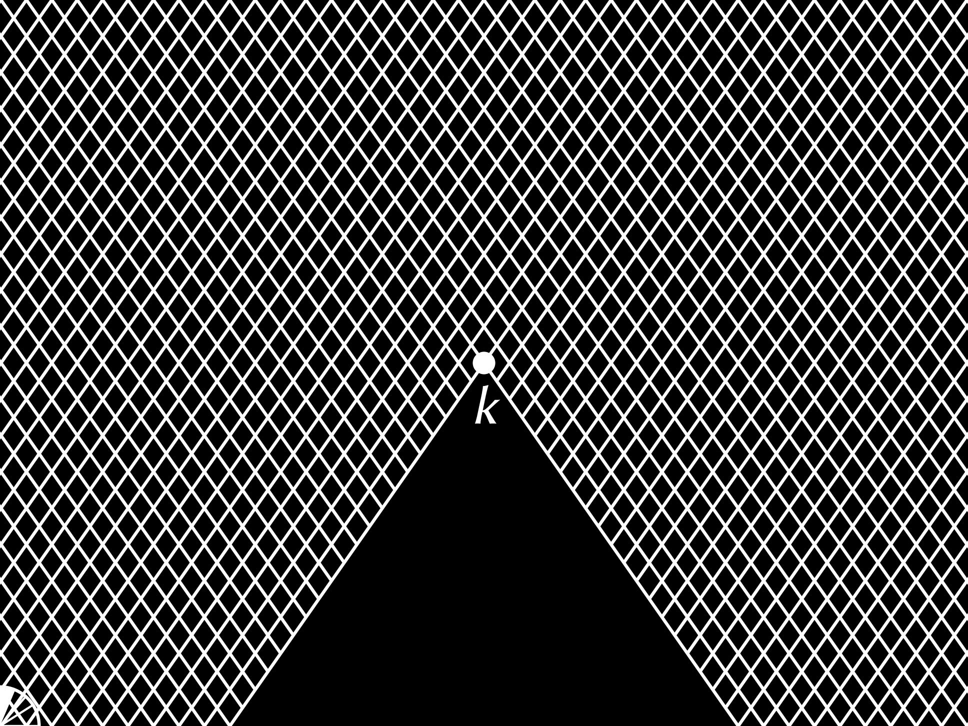
Call a model *concrete* whenever analogous equals equal.

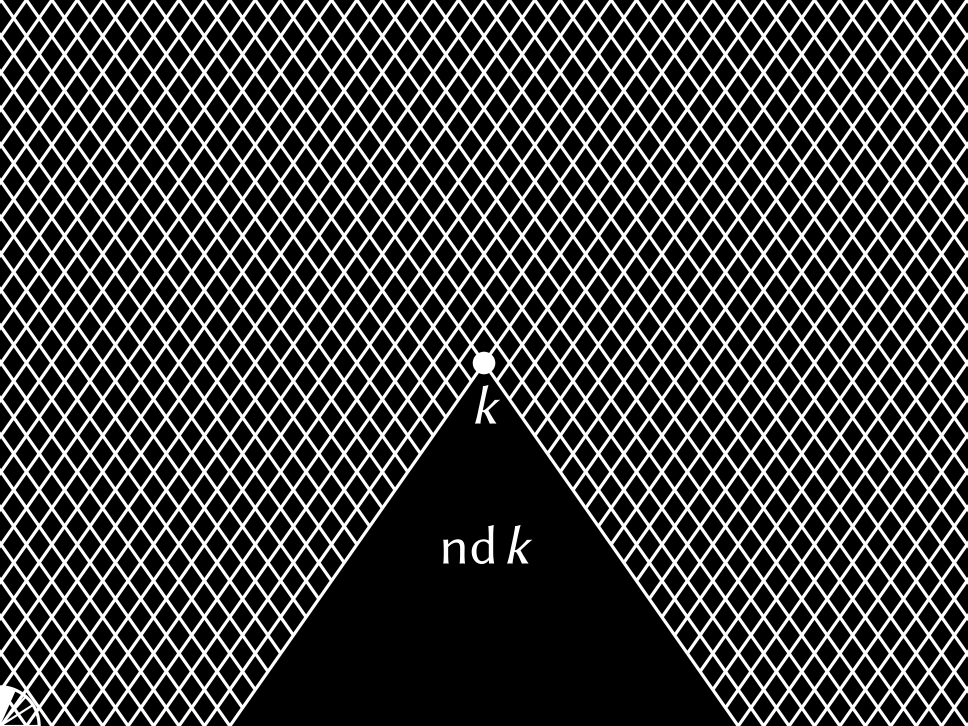
k

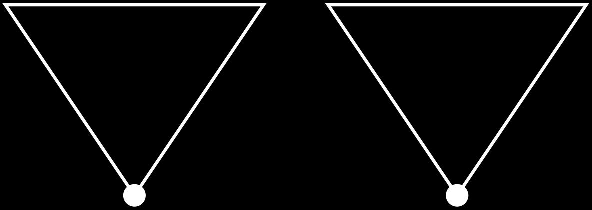

k

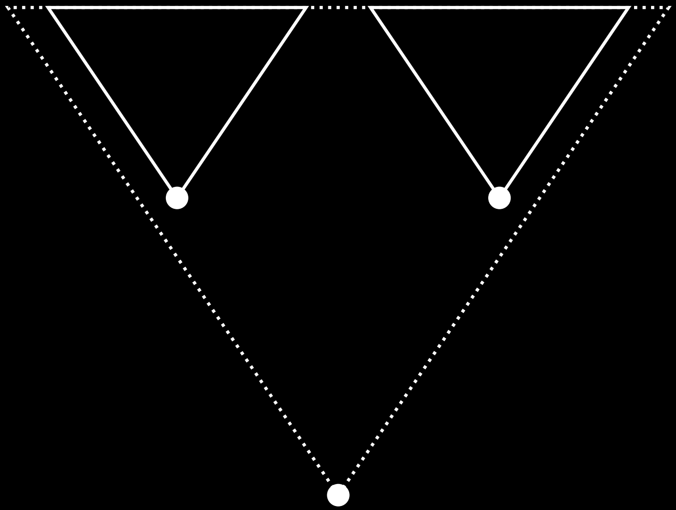


up k


k


k
nd k





$$\frac{K \Vdash A \vee B}{K \Vdash A \text{ or } K \Vdash B}$$

syntax

$$\frac{K \Vdash A \vee B}{K \Vdash A \text{ or } K \Vdash B}$$

semantics

syntax

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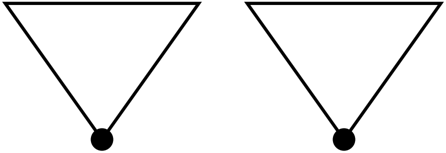
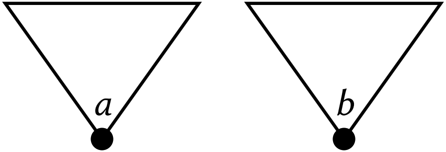


semantics

syntax

$$\frac{K \Vdash A \vee B}{K \Vdash A \text{ or } K \Vdash B}$$



semantics

syntax

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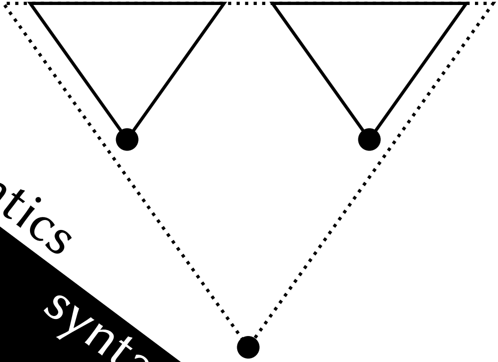
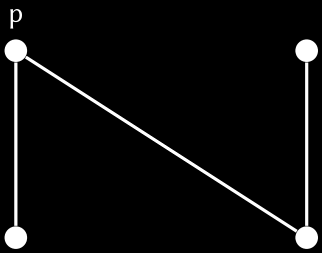


$$\frac{K \Vdash A \vee B}{K \Vdash A \text{ or } K \Vdash B}$$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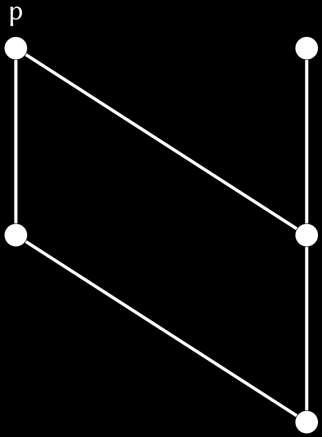
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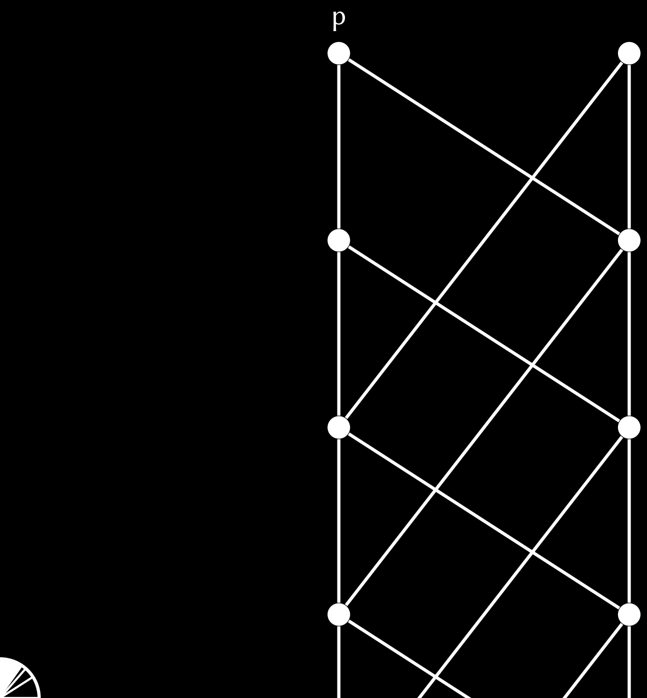
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There is a concrete & complete model to each finite set of variables.

There is a concrete & complete model to each finite set of variables X;
the universal model $\mathrm{U}(X)$.

1957 Rieger

1960 Nishimura

1968 de Jongh

1973 Urquhart

1975 Esakia and Grigolia

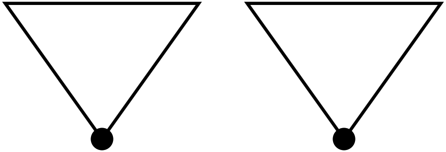
1978 Shehtman

1986 Bellissima

semantics

syntax

$$\frac{A \vee B}{A, B}$$



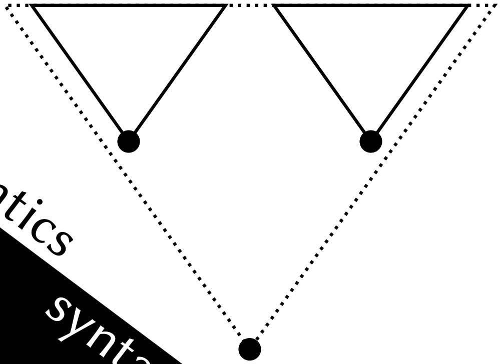
semantics

syntax

$$\frac{A \vee B}{A, B}$$

semantics

syntax



$$\frac{A \vee B}{A, B}$$

A / $\triangle$ admissible

if σA is derivable

$$A \,/\, \Delta \text{ admissible}$$

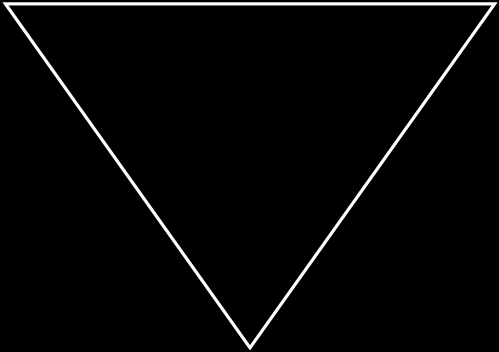
then σC is derivable, for some $C \in \Delta$

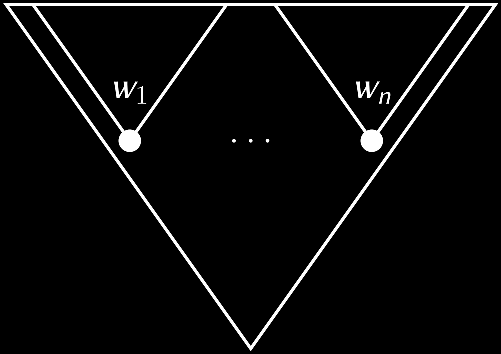
$$\frac{A \vee B}{A, B}$$

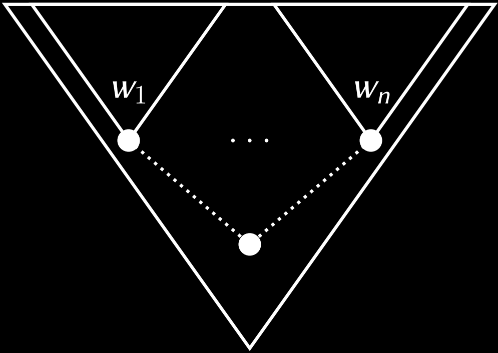
$$\frac{\neg C \to A \vee B}{\left(\neg C \to A\right) \vee \left(\neg C \to B\right)}$$

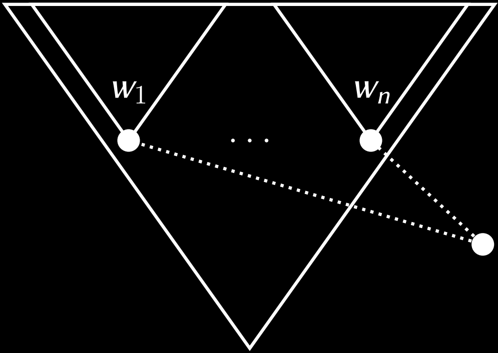
Weak Visser Rules

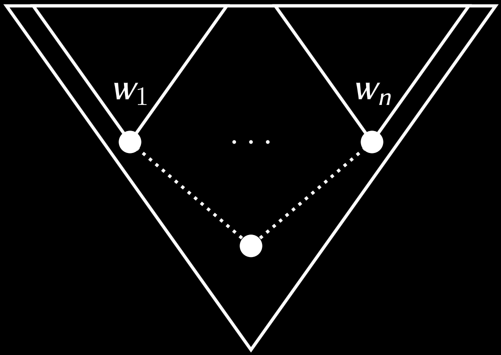
$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\bigvee \{(\bigvee \Delta \to A) \to \quad C \quad\}_{C \in \Delta}}$$




w_1
$\dots$
w_n

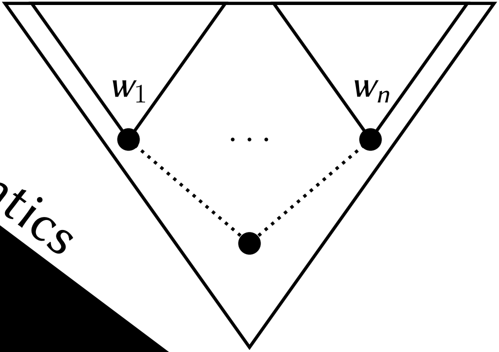

w_1
w_n
. . .

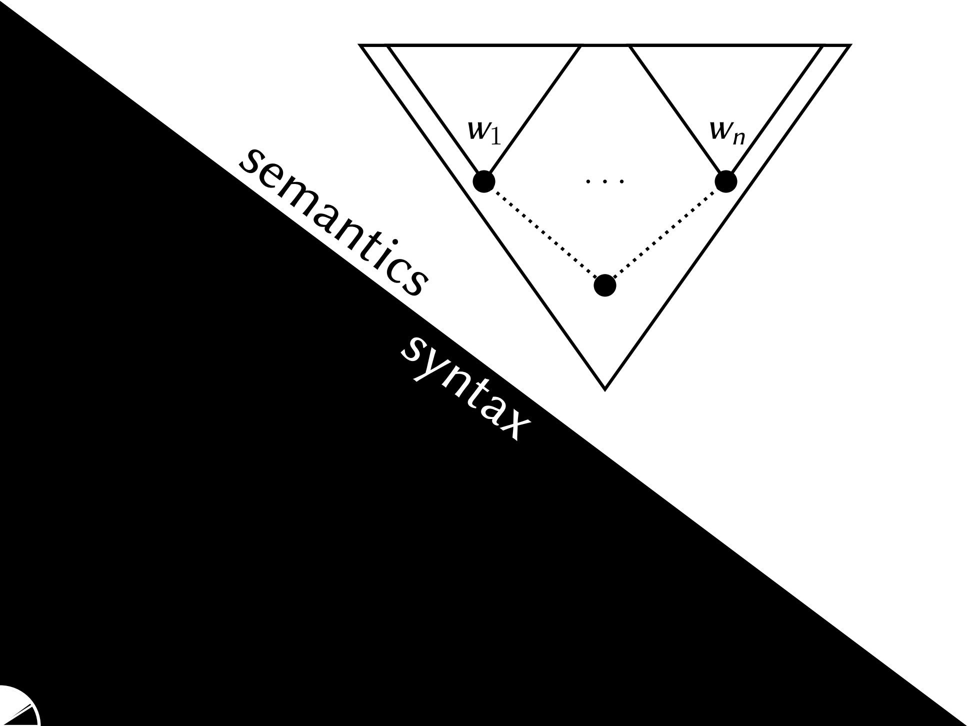

w_1
w_n
. . .

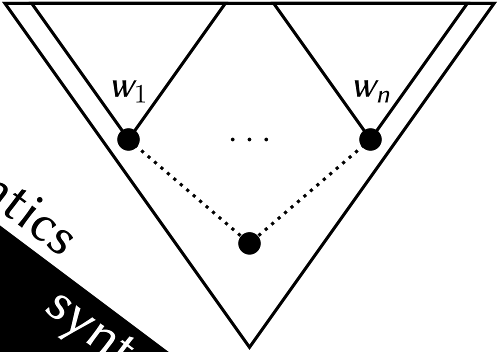


w_1
w_n

semantics



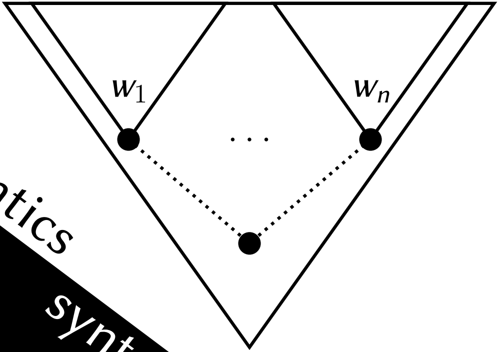

semantics
syntax
w_1
w_n
...



semantics

syntax

$$\frac{\left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \rightarrow \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \rightarrow \bigvee_{i=1}^{n} \mathsf{nd}\, w_i}{\bigvee_{j=1}^{n} \left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \rightarrow \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \rightarrow \mathsf{nd}\, w_j}$$



semantics

syntax

$$\frac{\left(\bigvee \Delta \to A \right) \to \bigvee \Delta}{\bigvee_{C \in \Delta} \left(\bigvee \Delta \to A \right) \to C}$$

A model K is *closed under* A/Δ
whenever
$K \Vdash A$ yields $K \Vdash C$ for some $C \in \Delta$.

Theorem (G, 2013):
Any concrete model satisfies the weak extension property iff it is closed under the weak Visser rules.

Corollary:
An intermediate logic with the finite model property has the weak extension property iff it admits the weak Visser rules.

Subframe logics are the logics that are closed under taking subframes.

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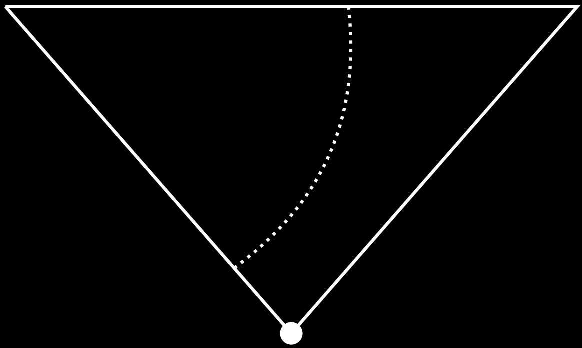

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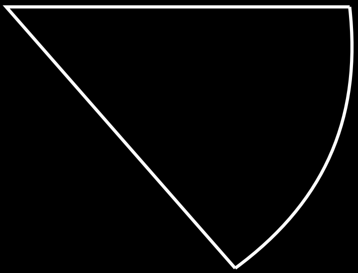


Theorem:
Subframe logics enjoy the weak extension property.

Theorem (Skura, 1989):
IPC is the only intermediate logic with the weak Visser rules and disjunction property.

Admissibility in BD_2

Logic of Depth n

$$\begin{aligned} \mathrm{bd}_0 &= \bot \\ \mathrm{bd}_{n+1} &= p_{n+1} \vee (p_{n+1} \to \mathrm{bd}_n). \end{aligned}$$

$$\mathrm{BD}_n := \mathrm{IPC} + \mathrm{bd}_n$$

BD_2 is the logic of frames of height at most two.

Does BD_2 admit the Weak Visser rules?

$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\bigvee \{(\bigvee \Delta \to A) \to \quad C \quad \}_{C \in \Delta}}$$

BD_2 *does* admit the Weak Visser rules.

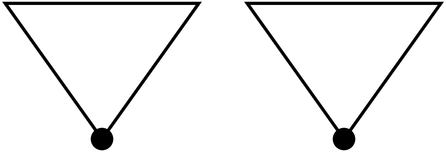
$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\bigvee \{(\bigvee \Delta \to A) \to C\}_{C \in \Delta}}$$

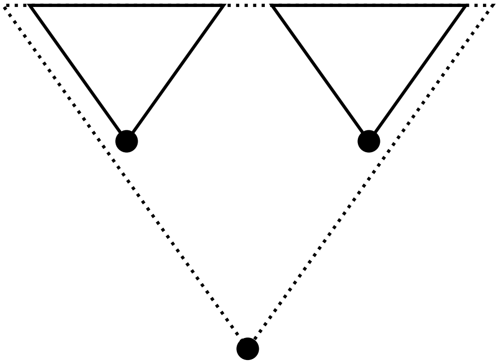
Does BD_2 enjoy the disjunction property?

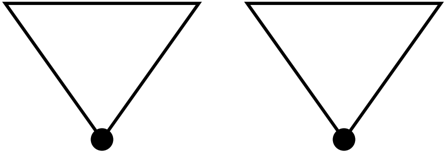
$$\frac{A \vee B}{A,\ B}$$

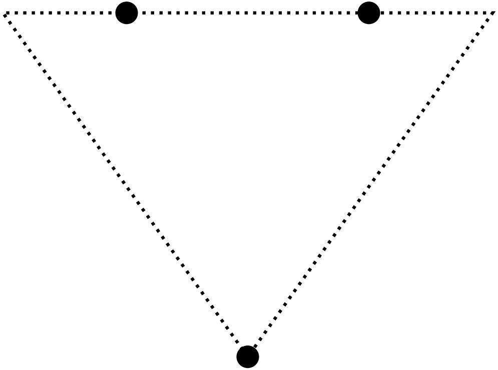
BD_2 *does not* enjoy
the disjunction property.

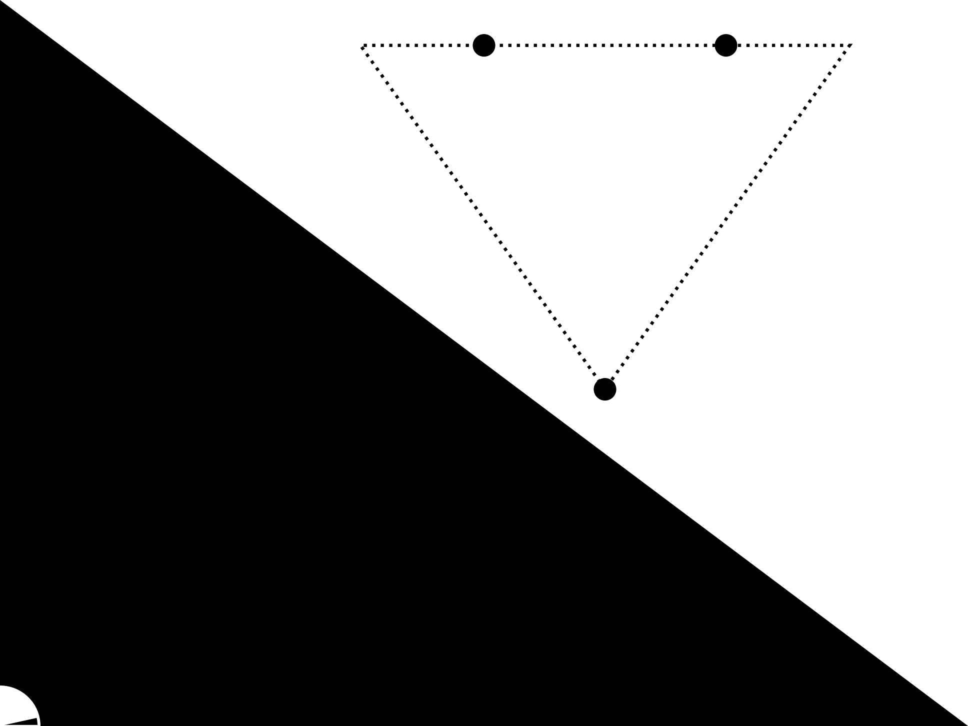
$$\frac{A \vee B}{A, B}$$

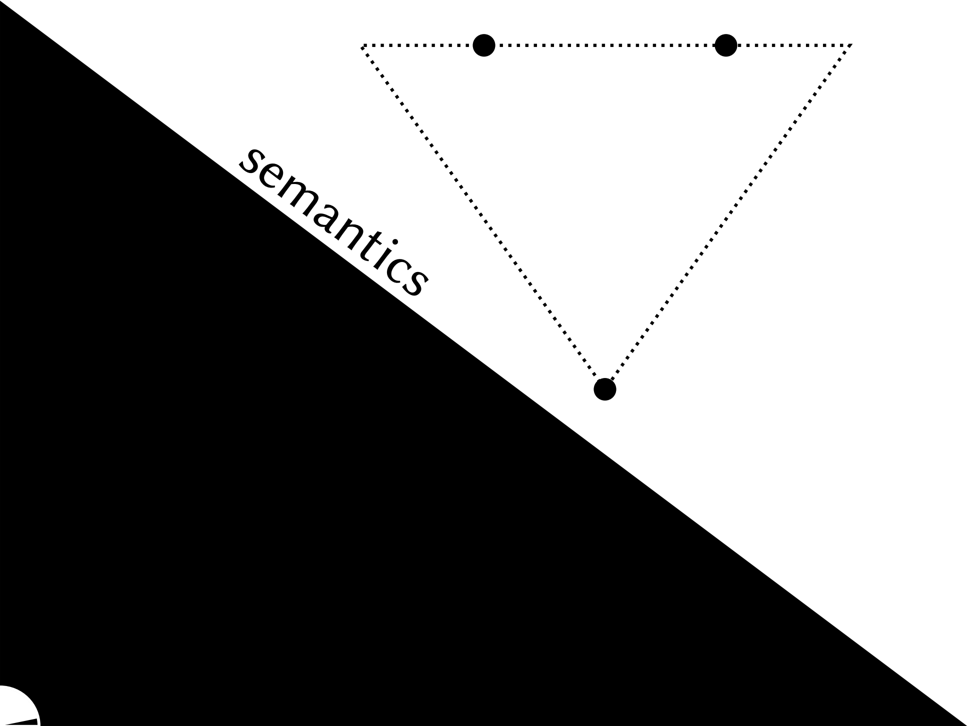


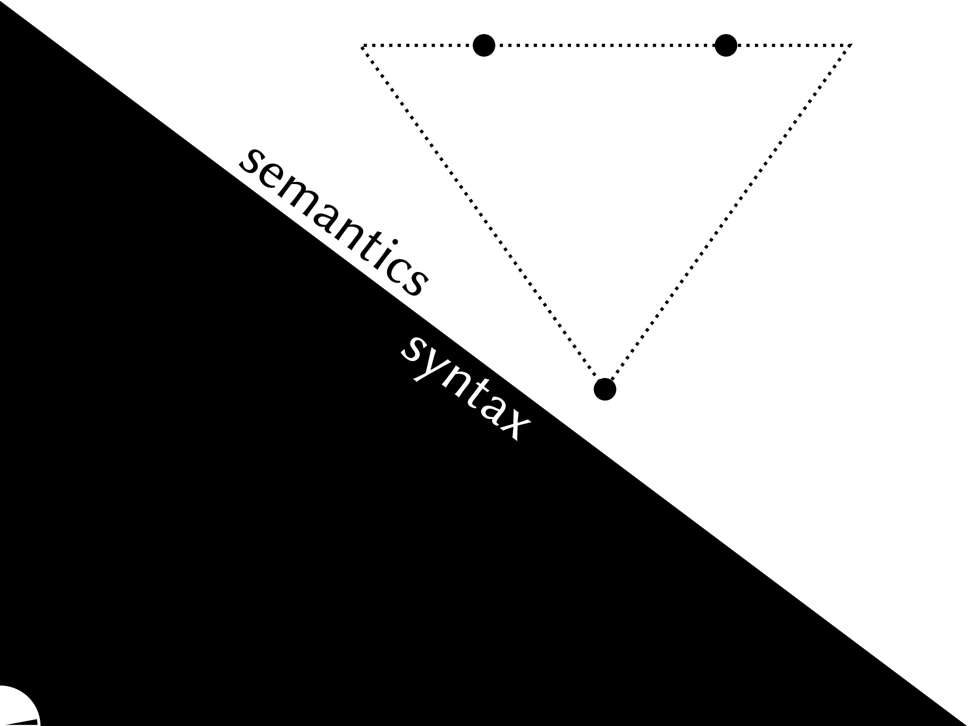


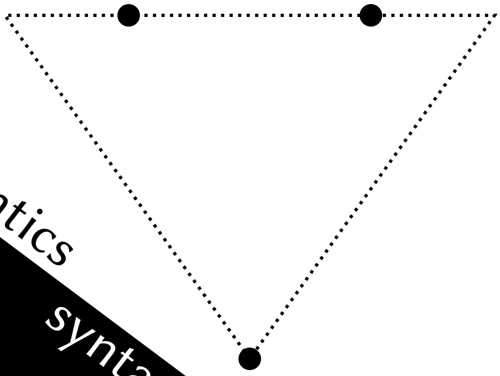







semantics


semantics
syntax



semantics

syntax

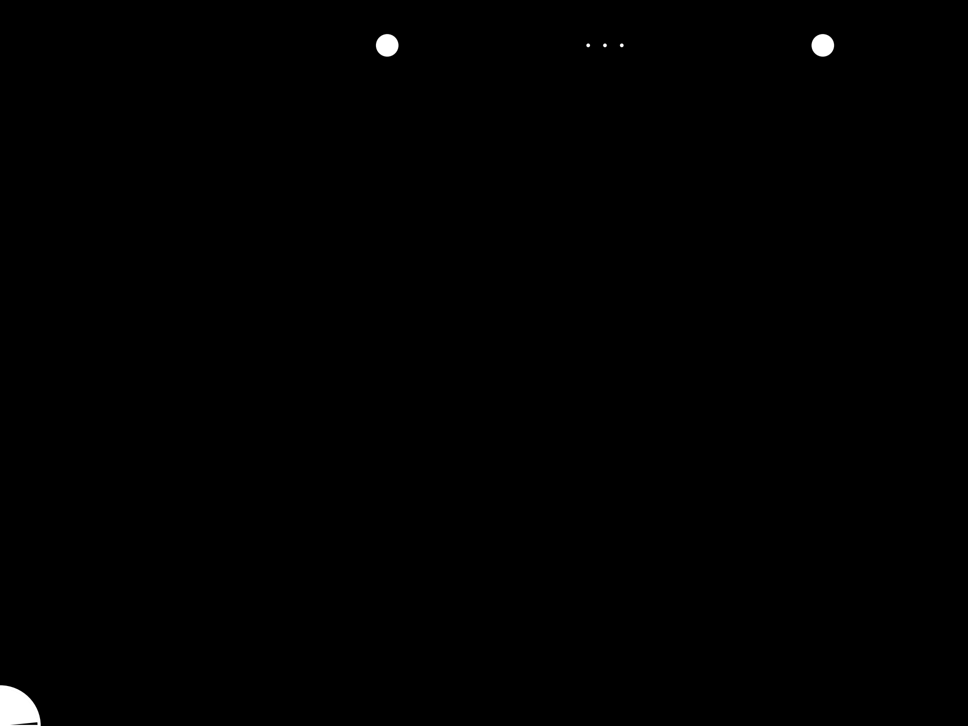
$$\frac{A \lor B}{\neg\neg A, \neg\neg B}$$

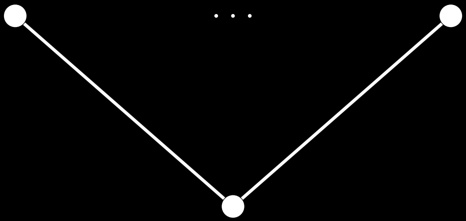
Does BD_2 enjoy the doubly-negated disjunction property?

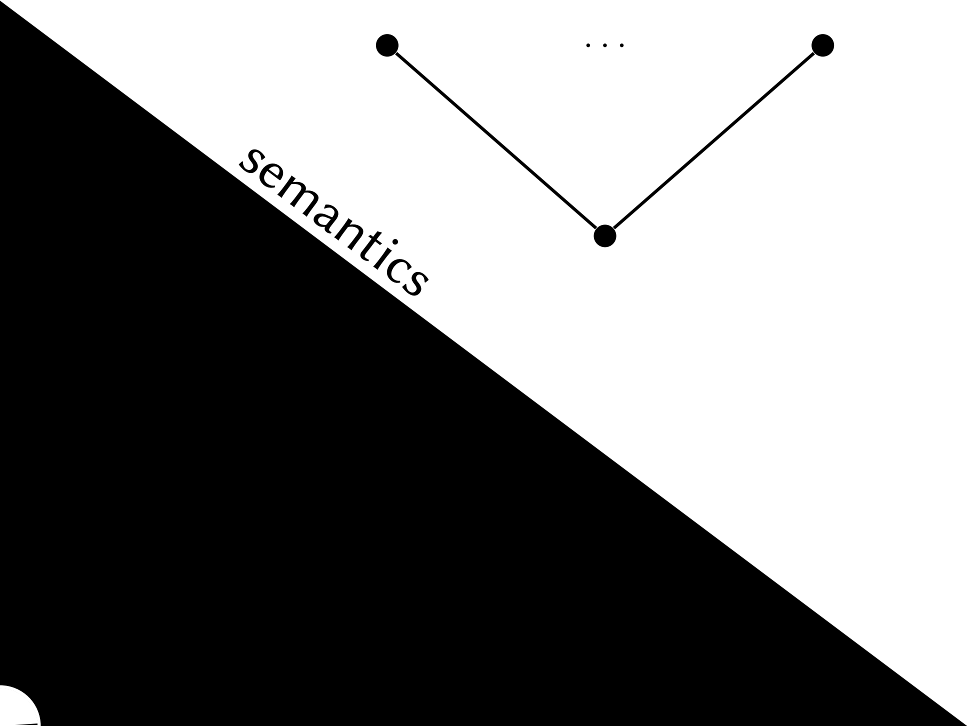
$$\frac{A \vee B}{\neg\neg A, \neg\neg B}$$

BD_2 *does* enjoy the doubly-negated disjunction property.

$$\frac{A \vee B}{\neg\neg A, \neg\neg B}$$



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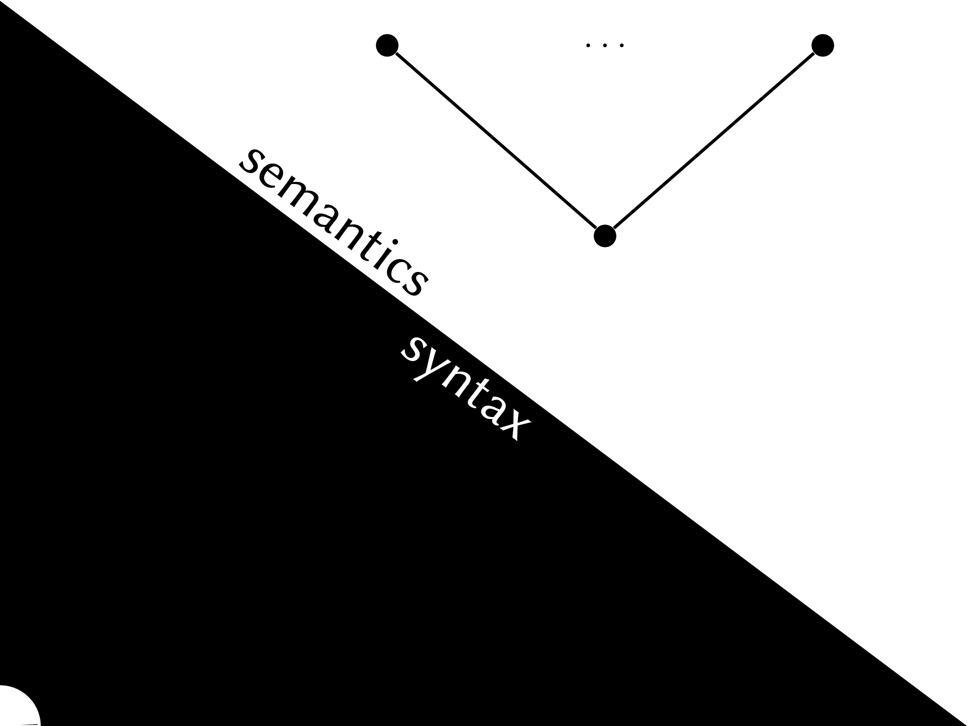
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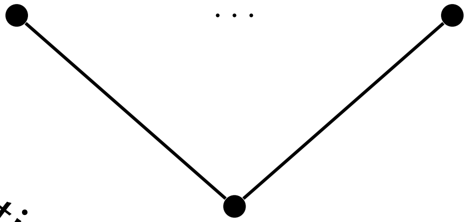

semantics

syntax

$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\{\neg\neg (\bigvee \Delta \to A) \to \; C \;\}_{C \in \Delta}}$$

Theorem (G 2013):
A concrete model is closed under all admissible rules of BD_2 iff it is closed under:

$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\{\neg\neg(\bigvee \Delta \to A) \to C\}_{C \in \Delta}}$$

Theorem (G 2013):
All admissible rules of BD_2
follow from:

$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\{\neg\neg(\bigvee \Delta \to A) \to C\}_{C \in \Delta}}$$

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