A Note on Extensions: Admissible Rules via Semantics

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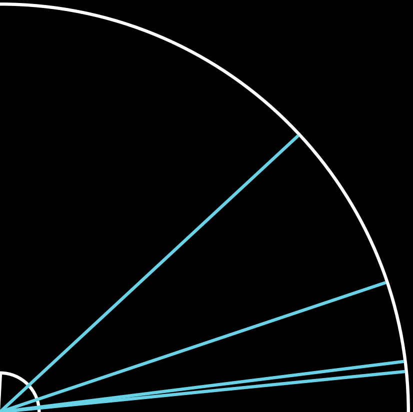
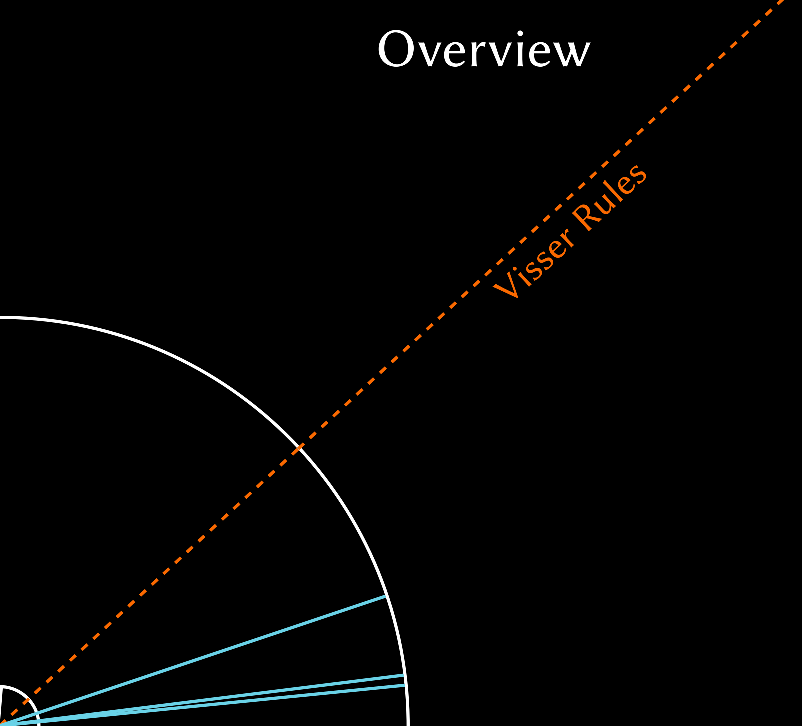
Utrecht University
9:20 — 9:45 Januari 7th 2013

Overview

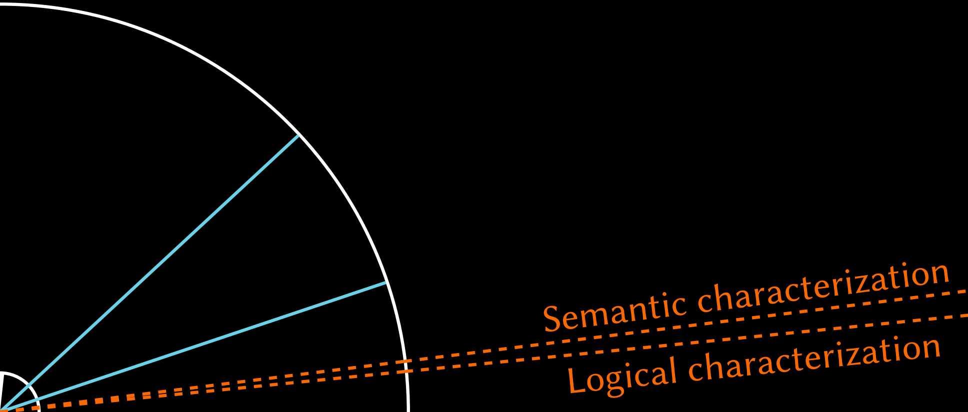
Overview



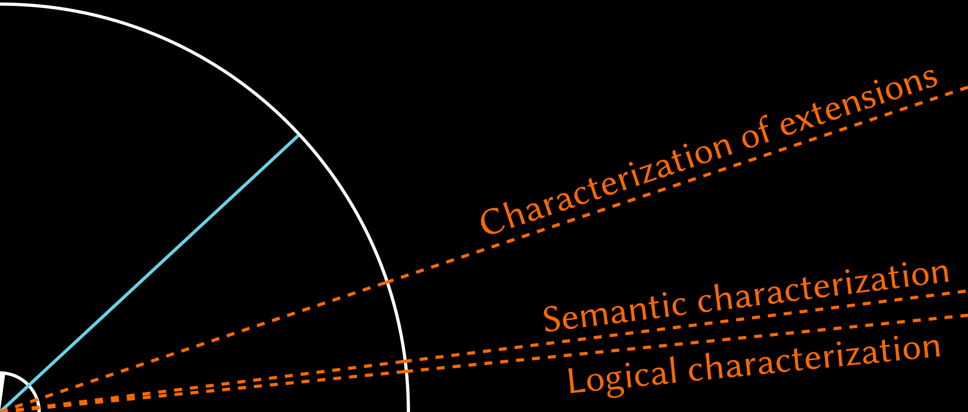
Overview



Overview



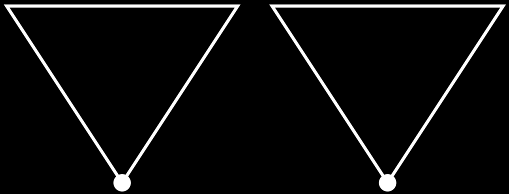
Overview


Characterization of extensions
Semantic characterization
Logical characterization

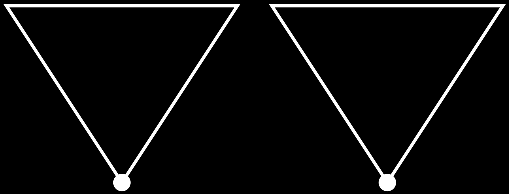
Disjunction Property

$$\frac{A \lor B \text{ derivable}}{A \text{ derivable or } B \text{ derivable}}$$

Semantic Counterpart

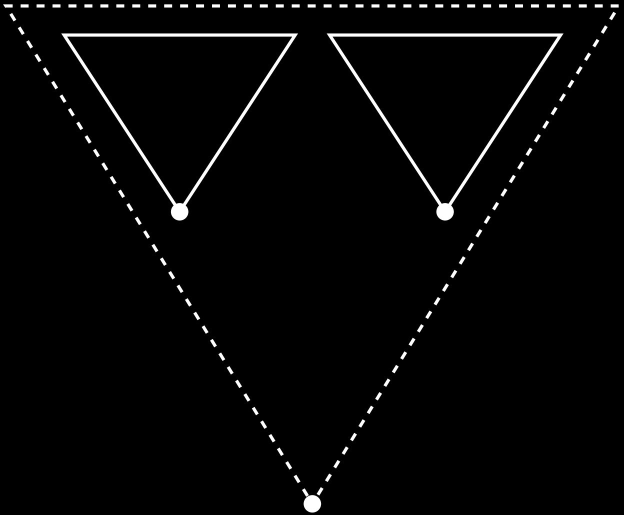


Semantic Counterpart



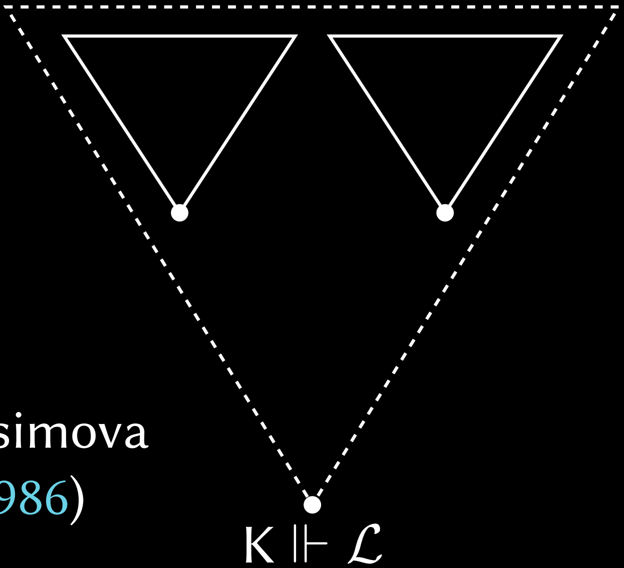
$K_1 \Vdash \mathcal{L}$ $\quad$ $K_2 \Vdash \mathcal{L}$

Semantic Counterpart



$K \Vdash \mathcal{L}$

Semantic Counterpart



Maksimova
(1986)

Conjecture of Łukasiewicz (1952):

There is no consistent theory strictly extending IPC closed under modes ponens, substitution with the disjunction property.

1952

Łukasiewicz

Kreisel and Putnam

1952

1957

Łukasiewicz

Kreisel and Putnam

1952

1957

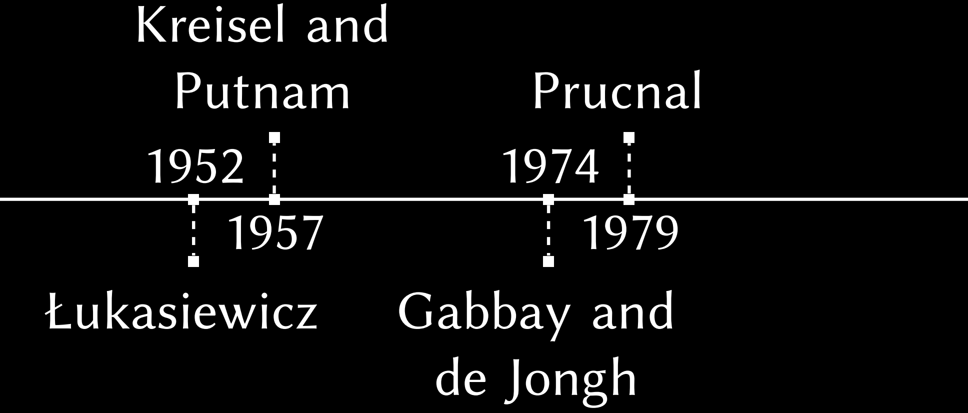
Łukasiewicz

$$(\neg A \to D \vee E) \to (\neg A \to D) \vee (\neg A \to E)$$

1952
Łukasiewicz

1957
Kreisel and Putnam

1974
Gabbay and de Jongh


Kreisel and Putnam
Prucnal
1952
1974
1957
1979
Łukasiewicz
Gabbay and de Jongh

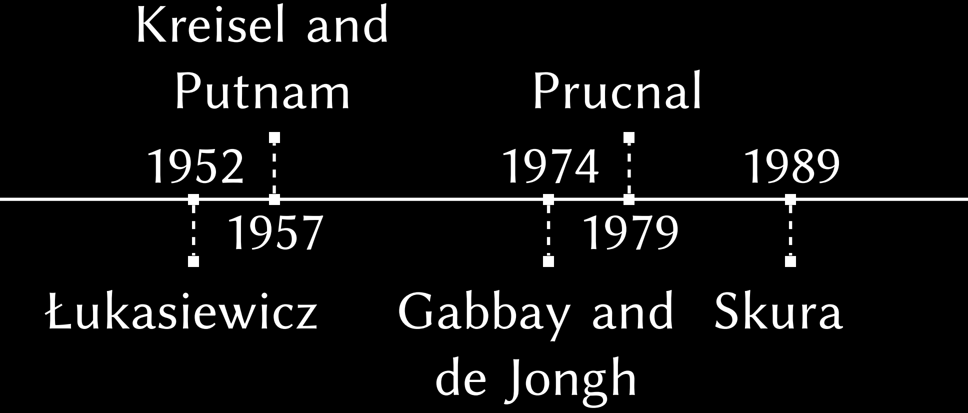
Kreisel and Putnam — 1957

Prucnal — 1979

Łukasiewicz — 1952

Gabbay and de Jongh — 1974

$$\frac{\neg A \to D \vee E}{(\neg A \to D) \vee (\neg A \to E)}$$


Kreisel and Putnam
Prucnal
1952
1974
1989
1957
1979
Łukasiewicz
Gabbay and de Jongh
Skura

Kreisel and Putnam 1957

Prucnal 1979

1952 Łukasiewicz

1974 Gabbay and de Jongh

1989 Skura

$$\frac{\bigwedge_{i=1}^{n}(A_i \to B_i) \to \bigvee_{i=1}^{n} A_i}{\bigwedge_{i=1}^{n}(A_i \to B_i) \to A_j \text{ for some } j \in \{1, \ldots, n\}}$$

S/T admissible

A^σ is derivable for each $A \in \mathbf{S}$

$\mathbf{S}/\mathbf{T}$ admissible

B^σ is derivable for some $B \in \mathbf{T}$

$$\frac{D_1 \lor D_2}{\{D_1, D_2\}}$$

$$\frac{\bigvee \Delta}{\{\, D \mid D \in \Delta \,\}}$$

$$\frac{\neg A \to \bigvee \Delta}{\{\, \neg A \to D \mid D \in \Delta \,\}}$$

$$\frac{\neg A \to \bigvee \Delta}{\{\, \neg A \to D \mid D \in \Delta \text{ or } D = A \,\}}$$

$$\frac{(A \to \bot) \to \bigvee \Delta}{\{\ (A \to \bot) \to D \mid D \in \Delta \text{ or } D = A\ \}}$$

$$\frac{\bigwedge (A_i \to B_i) \to \bigvee \Delta}{\{ \bigwedge (A_i \to B_i) \to D \mid D \in \Delta \text{ or } D = A_j \}}$$

Visser Rules

$$\frac{\bigwedge (A_i \to B_i) \to \bigvee \Delta}{\{\ \bigwedge (A_i \to B_i) \to D \mid D \in \Delta \text{ or } D = A_j\ \}}$$

Skura Rules

$$\frac{\bigwedge (A_i \to B_i) \to \bigvee A_i}{\{ \bigwedge (A_i \to B_i) \to D \mid D = A_j \}}$$

1975

Friedman

1975

Friedman

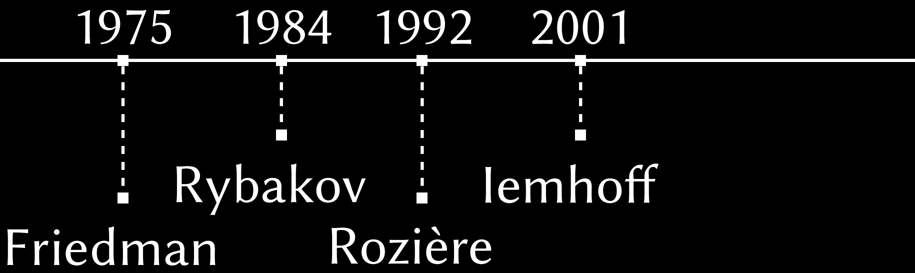
Can one decide whether
a rule is admissible for IPC?

1975 1984

Rybakov

Friedman

One can decide whether
a rule is admissible for IPC.



The Visser rules
axiomatize admissibility
in IPC.

1975 1984 1992 2001 2012

Friedman

Rybakov

Rozière

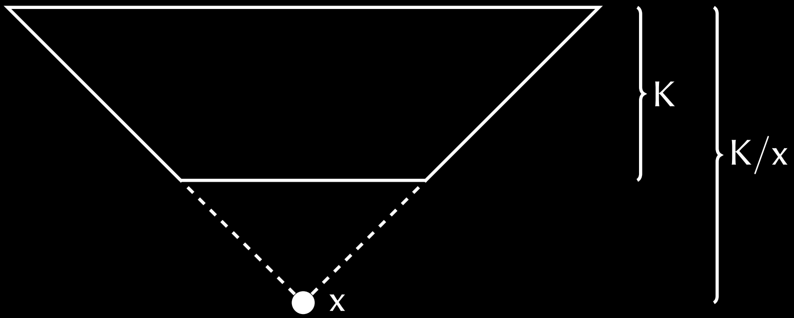
Iemhoff

Goudsmit and Iemhoff

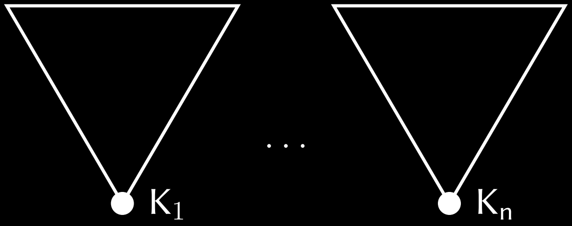
The de Jongh rules
axiomatize admissibility
in Gabbay–de Jongh logics.

Extension

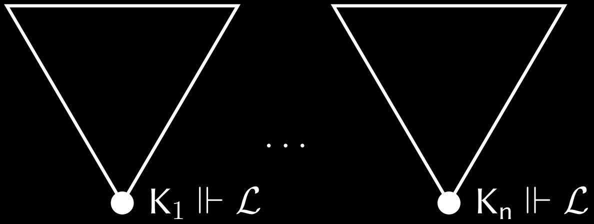
Extension



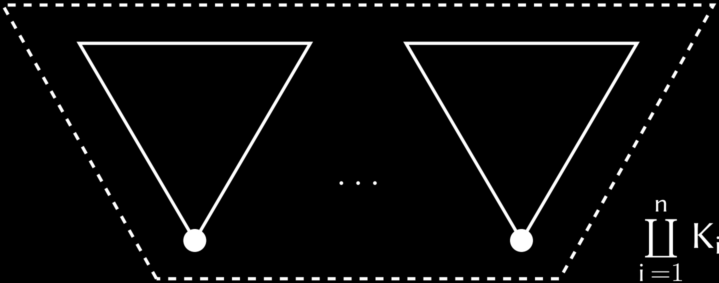
Extension Property



Extension Property

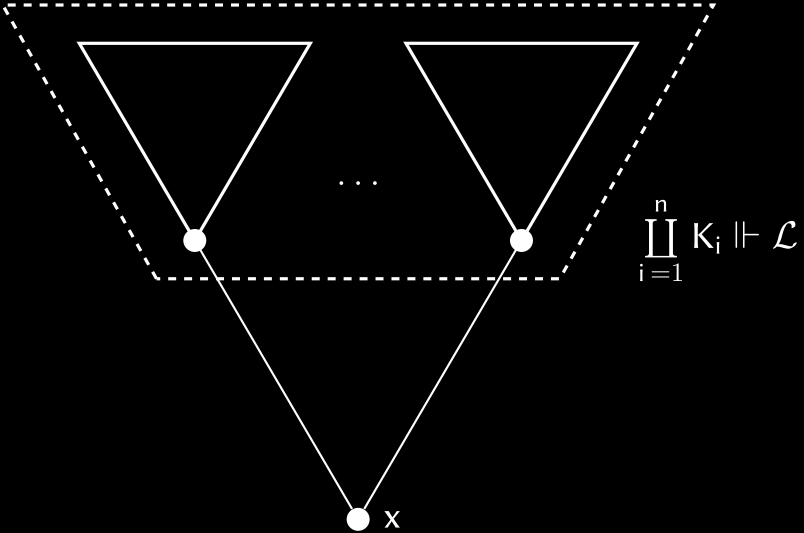


Extension Property

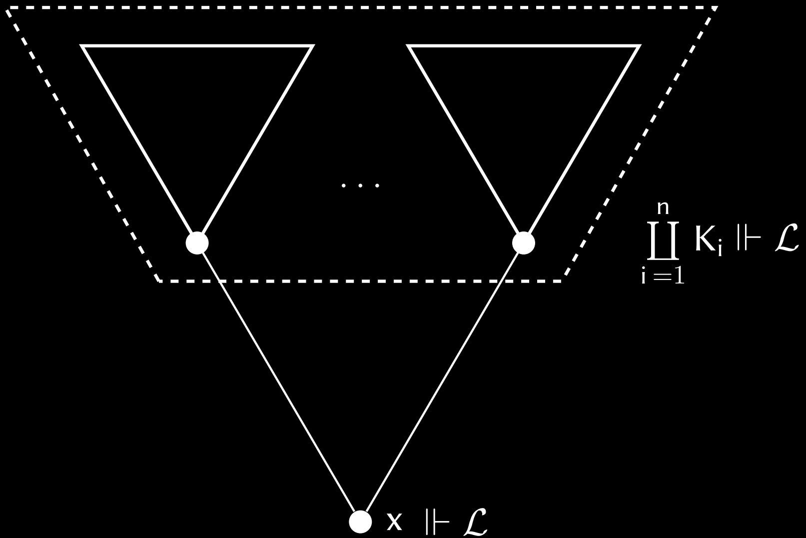


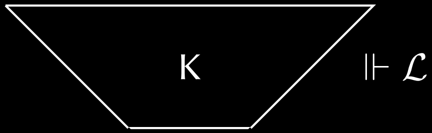
$$\coprod_{i=1}^{n} K_i \Vdash \mathcal{L}$$

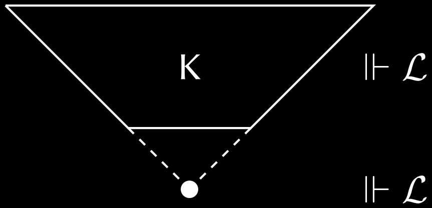
Extension Property

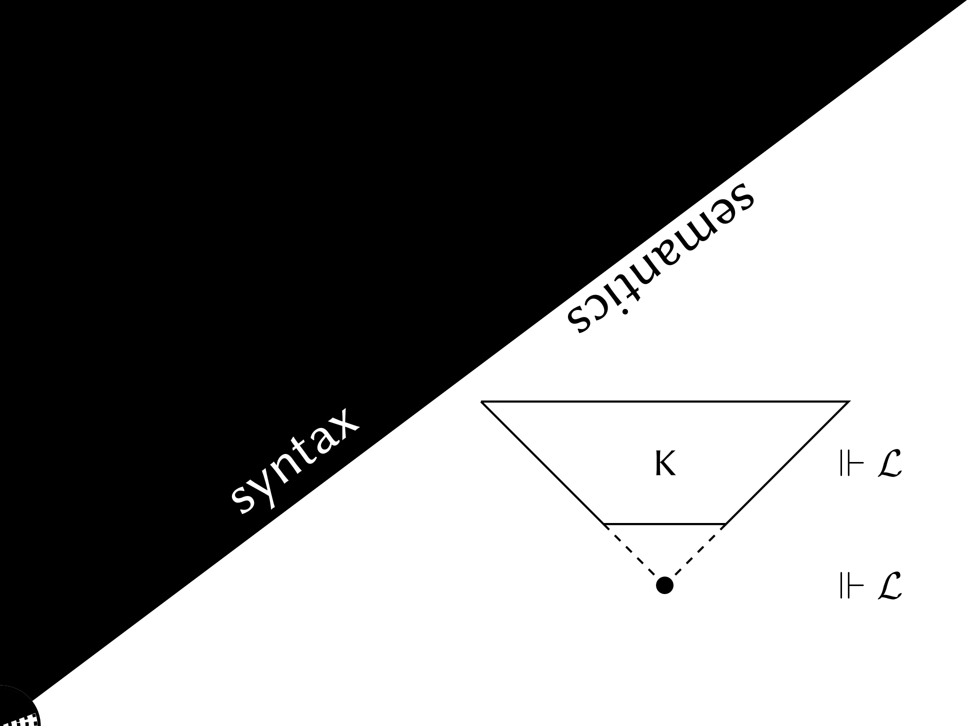


Extension Property




K
⊩ $\mathcal{L}$


K
⊩ $\mathcal{L}$
⊩ $\mathcal{L}$

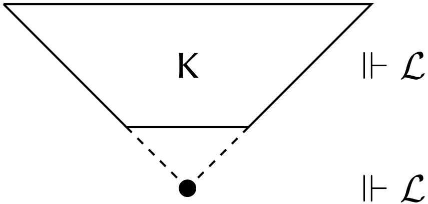


syntax
semantics
K
⊩ $\mathcal{L}$
⊩ $\mathcal{L}$

$$\frac{\left\{A \to B \;\middle|\; \begin{array}{l} K \Vdash A \to B \\ K \nVdash A \end{array}\right\} \vdash_{\mathcal{L}} \bigvee \Delta}{K \Vdash D \text{ for some } D \in \Delta}$$

semantics

syntax



Skura via Extensions

$$\frac{\bigwedge_{i=1}^{n}(A_i \to B_i) \to \bigvee_{i=1}^{n} A_i}{\{\ \bigwedge_{i=1}^{n}(A_i \to B_i) \to A_j \mid j = 1, \ldots, n\ \}}$$

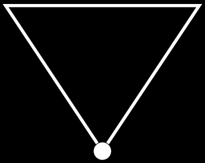
Skura via Extensions

$$\frac{\bigwedge_{i=1}^{n}(A_i \to B_i) \to \bigvee_{i=1}^{n} A_i}{\{\ \bigwedge_{i=1}^{n}(A_i \to B_i) \to A_j \mid j = 1, \ldots, n\ \}}$$

Skura via Extensions

$K_1 \Vdash \bigwedge_{i=1}^{n} A_i \to B_i$

$K_1 \nVdash A_1$

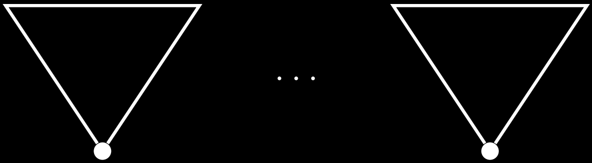


$$\frac{\bigwedge_{i=1}^{n}(A_i \to B_i) \to \bigvee_{i=1}^{n} A_i}{\{\ \bigwedge_{i=1}^{n}(A_i \to B_i) \to A_j \mid j = 1, \ldots, n\ \}}$$

Skura via Extensions

$K_1 \Vdash \bigwedge_{i=1}^{n} A_i \to B_i$ $\qquad$ $K_n \Vdash \bigwedge_{i=1}^{n} A_i \to B_i$

$K_1 \nVdash A_1$ $\qquad$ $K_n \nVdash A_n$

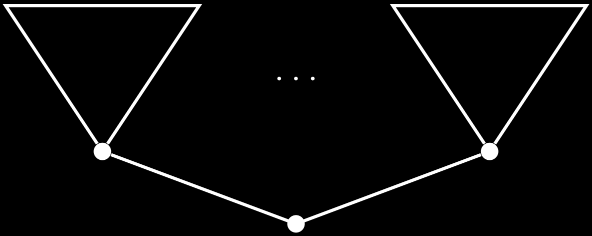


$$\frac{\bigwedge_{i=1}^{n}(A_i \to B_i) \to \bigvee_{i=1}^{n} A_i}{\{\ \bigwedge_{i=1}^{n}(A_i \to B_i) \to A_j \mid j = 1, \ldots, n\ \}}$$

Skura via Extensions

$K_1 \Vdash \bigwedge_{i=1}^{n} A_i \to B_i$ $\qquad$ $K_n \Vdash \bigwedge_{i=1}^{n} A_i \to B_i$

$K_1 \nVdash A_1$ $\qquad$ $K_n \nVdash A_n$



$$\frac{\bigwedge_{i=1}^{n}(A_i \to B_i) \to \bigvee_{i=1}^{n} A_i}{\{\ \bigwedge_{i=1}^{n}(A_i \to B_i) \to A_j \mid j = 1, \ldots, n\ \}}$$

Semantic Characterization

Any intermediate logic with the disjunction property
admits the de Jongh rule
iff
it has the extension property.

Semantic Characterization

Any intermediate logic with the disjunction property admits the n^{th} de Jongh rule iff it has the n^{th} extension property.

Semantic Characterization
Goudsmit (2013)

Any extension of minimal logic with the disjunction property admits the n^{th} de Jongh rule
iff
it has the n^{th} extension property.

Logical Characterization
Goudsmit and Iemhoff (2012)

The n^{th} de Jongh rule
axiomatises admissibility
of the n^{th} Gabbay—de Jongh logic.

The next step:

Finding an axiomatisation of admissibility over minimal logic.

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Finding an axiomatisation of admissibility over minimal logic.

References I

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