On the admissible rules of Gabbay–de Jongh logics

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$A \vee B$ derivable

$$\frac{A \vee B \text{ derivable}}{A \text{ or } B \text{ derivable}}$$

$$\frac{\neg C \to A \vee B}{(\neg C \to A) \vee (\neg C \to B)}$$

S/T admissible

A^σ is derivable for each $A \in \mathbf{S}$

$\mathbf{S}/\mathbf{T}$ admissible

B^σ is derivable for some $B \in \mathbf{T}$

$$\frac{\{\, p \vee q \,\}}{\{\, p, q \,\}}$$

$$\frac{p \lor q}{\{\, p, q \,\}}$$

$$\frac{\Rightarrow p \vee q}{\{\ \Rightarrow p,\ \Rightarrow q\ \}}$$

$$\frac{\Rightarrow p \ , \quad q}{\{\ \Rightarrow p,\ \Rightarrow q\ \}}$$

$$\frac{\Rightarrow \Delta}{\{\ \Rightarrow \Pi\ \}_{\Pi \in \Delta}}$$

$$\frac{\neg a \Rightarrow \Delta}{\{\ \neg a \Rightarrow \Pi\ \}_{\Pi \in \Delta}}$$

$$\frac{a \to \bot \Rightarrow \Delta}{\{\, a \to \bot \Rightarrow \Pi \,\}_{\Pi \in \Delta}}$$

$$\{ a_i \to \bot \}_{i=1}^{n}$$

$$\vdots$$

$$\frac{\Gamma \Rightarrow \Delta}{\{ \Gamma \Rightarrow \Pi \}_{\Pi \in \Delta}}$$

$$\{\ a_i \to b_i\ \}_{i=1}^{n}$$

$$\Big|$$

$$\Gamma \Rightarrow \Delta$$

$$\{\ \Gamma \Rightarrow \Pi\ \}_{\Pi \in \Delta} \cup \{\ \Gamma \Rightarrow a_i\ \}_{i=1}^{n}$$

$$\frac{\Gamma \Rightarrow \Delta}{\{\, \Gamma \Rightarrow \Pi \,\}_{\Pi \in \Delta} \cup \{\, \Gamma \Rightarrow \Pi \,\}_{\Pi \in \Gamma^{a}}}$$

Visser Rules

$$\frac{\Gamma \Rightarrow \Delta}{\{\ \Gamma \Rightarrow \Pi\ \}_{\Pi\, \in\, \Delta\, \cup\, \Gamma^{a}}}$$

Friedman (1975)

Can one decide whether a rule is admissible for IPC?

Rybakov (1984)

Yes

Do the Visser rules characterize admissibility?

Iemhoff (2001) & Rozière (1992)

Yes

What about intermediate logics?

Iemhoff (2005)

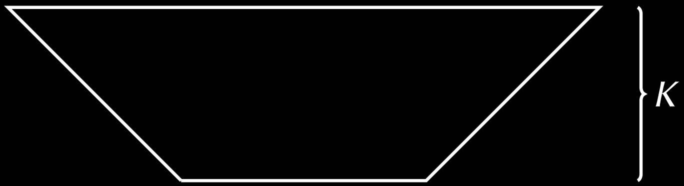
When the Visser rules are admissible, they characterize.

And when they are not?

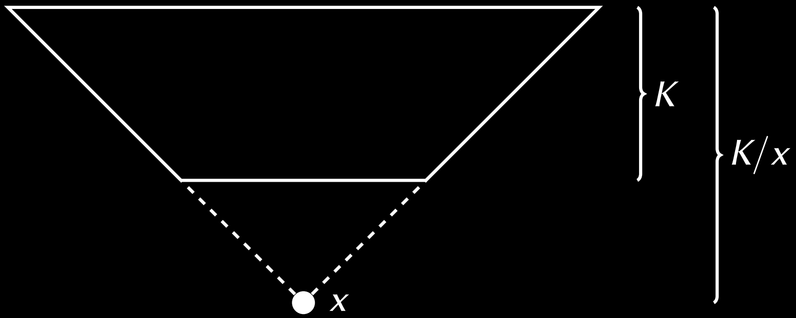
Gabbay and de Jongh (1974):

Logic of at most $(n + 1)$-fold branching finite trees

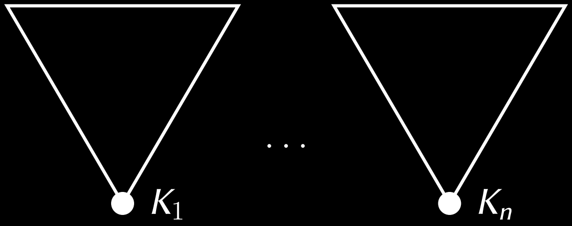
Extension



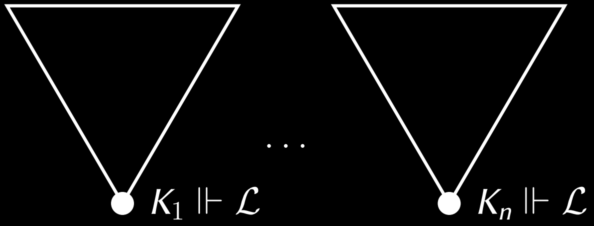
Extension



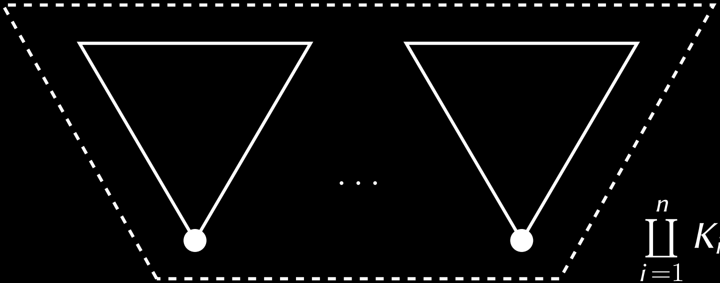
Extension Property



Extension Property

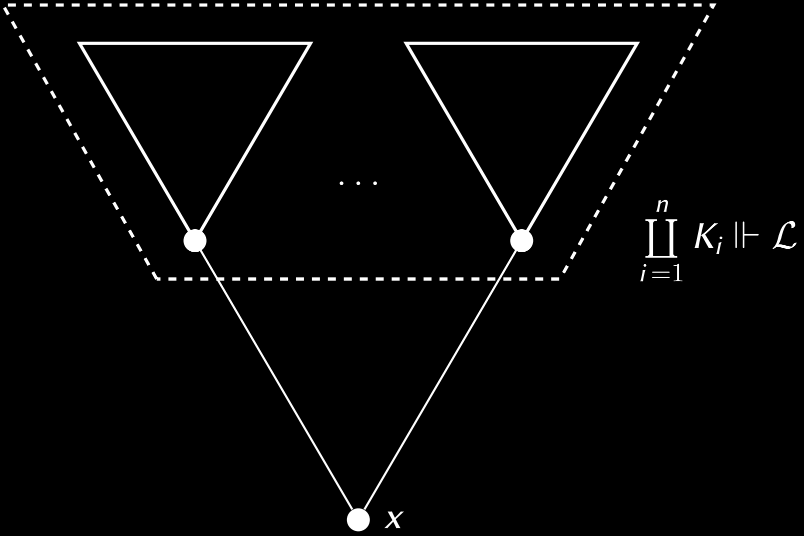


Extension Property

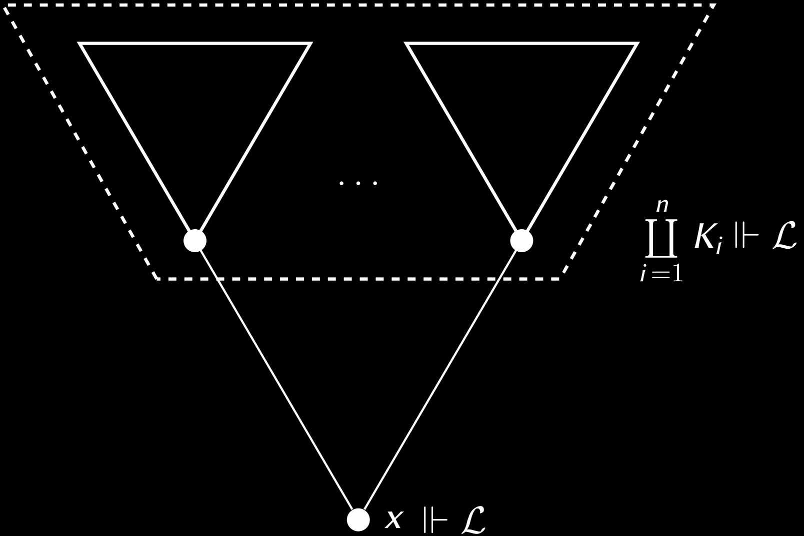


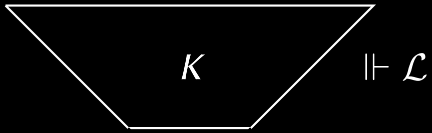
$$\coprod_{i=1}^{n} K_i \Vdash \mathcal{L}$$

Extension Property



Extension Property




K
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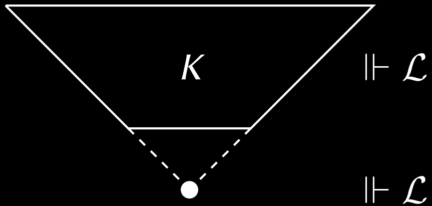
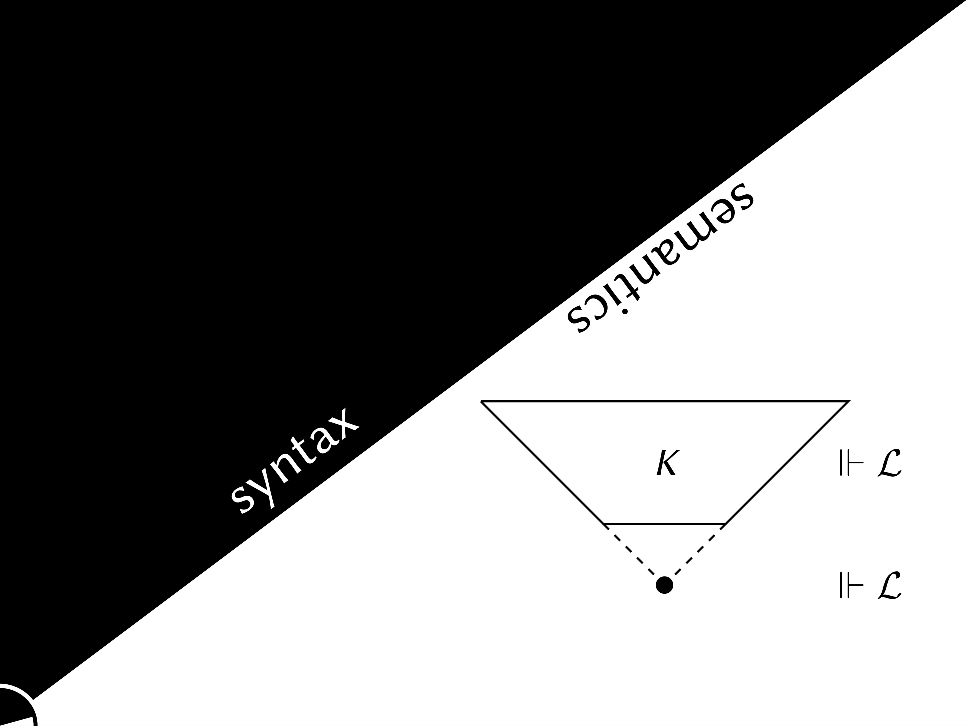

K
$\Vdash \mathcal{L}$
$\Vdash \mathcal{L}$

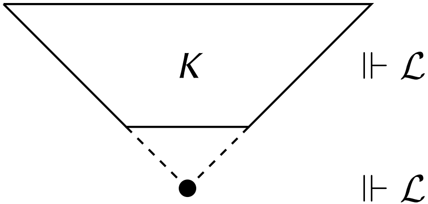

syntax
semantics
K
$\Vdash \mathcal{L}$
$\Vdash \mathcal{L}$

If $\left\{ A \to B \;\middle|\; \begin{array}{l} K \Vdash A \to B \\ K \nVdash A \end{array} \right\} \vdash_{\mathcal{L}} \bigvee \Delta$

then Δ intersects $\mathrm{Th}\, K$

syntax

semantics



de Jongh Rules

$$\frac{\Gamma \Rightarrow \Delta}{\{\ \Gamma \Rightarrow \Pi\ \}_{\Pi\, \in\, \Delta \cup \mathcal{U}}}$$

de Jongh Rules

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covers Γ^{a} & has *n* elements

Intermediate logic with DP:

admits n^{th} de Jongh rule

iff

has n^{th} extension property

Goudsmit and Iemhoff (2012)

$(n+1)^{\text{th}}$ de Jongh rule
characterizes admissibility
of n^{th} Gabby–de Jongh Logic

Goudsmit and Iemhoff (2012)

$(n+1)^{\text{th}}$ de Jongh rule
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& any extension thereof

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References I

- Friedman, Harvey (1975). "One Hundred and Two Problems in Mathematical Logic". In: *The Journal of Symbolic Logic* 40.2, pp. 113–129. ISSN: 00224812. JSTOR: 2271891.
- Gabbay, Dov M. and Dick H.J. de Jongh (1974). "A Sequence of Decidable Finitely Axiomatizable Intermediate Logics with the Disjunction Property". In: *The Journal of Symbolic Logic* 39.1, pp. 67–78. ISSN: 00224812. JSTOR: 2272344.
- Goudsmit, Jeroen P. and Rosalie Iemhoff (2012). "On unification and admissible rules in Gabbay-de Jongh logics". In: *Logic Group Preprint Series* 297, pp. 1–18. ISSN: 0929-0710. URL: http://phil.uu.nl/preprints/lgps/number/297.
- Iemhoff, Rosalie (2001). "On the Admissible Rules of Intuitionistic Propositional Logic". In: *The Journal of Symbolic Logic* 66.1, pp. 281–294. ISSN: 00224812. JSTOR: 2694922.

References II

Iemhoff, Rosalie (2005). “Intermediate Logics and Visser’s Rules”. In: *Notre Dame Journal of Formal Logic* 46.1, pp. 65–81. DOI: 10.1305/ndjfl/1107220674.

Rozière, Paul (1992). “Règles admissibles en calcul propositionnel intuitionniste”. PhD thesis. Université de Paris VII.

Rybakov, Vladimir V. (1984). “A criterion for admissibility of rules in the model system S4 and the intuitionistic logic”. In: *Algebra and Logic* 23 (5), pp. 369–384. ISSN: 0002-5232. DOI: 10.1007/BF01982031.