

Geschiedenis van de Logica

Logica in Informatica

Jeroen Goudsmit

Universiteit Utrecht

maandag 27 juni 2011

Inhoud



Overzicht

Herschrijven

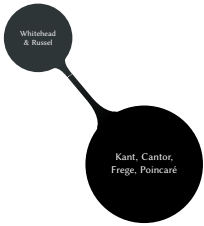
Typen

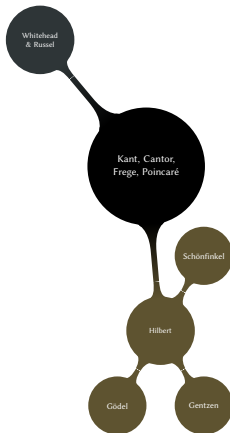
Toepassing

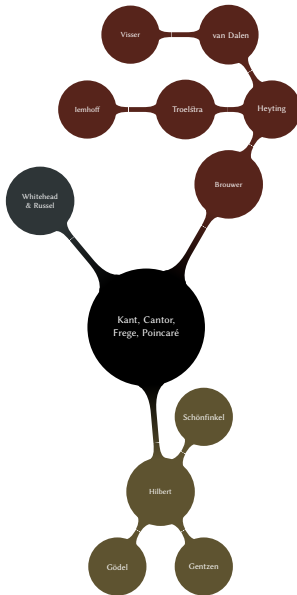
Overzicht

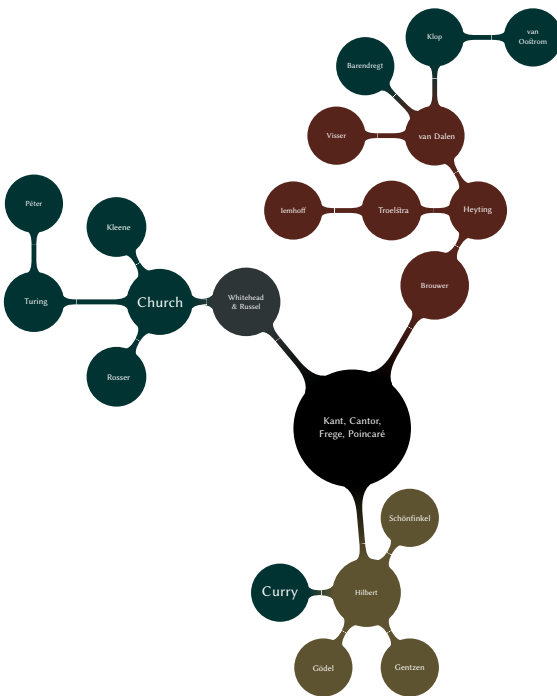


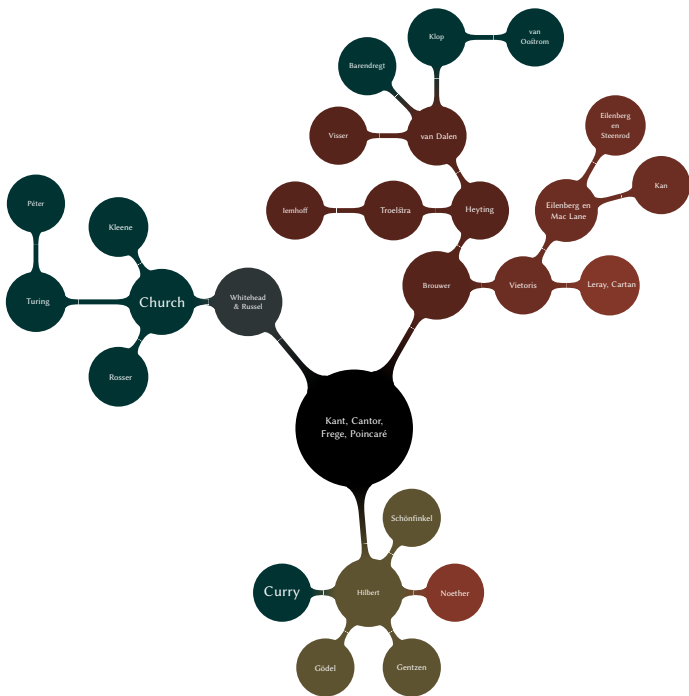
Kant, Cantor,
Frege, Poincaré

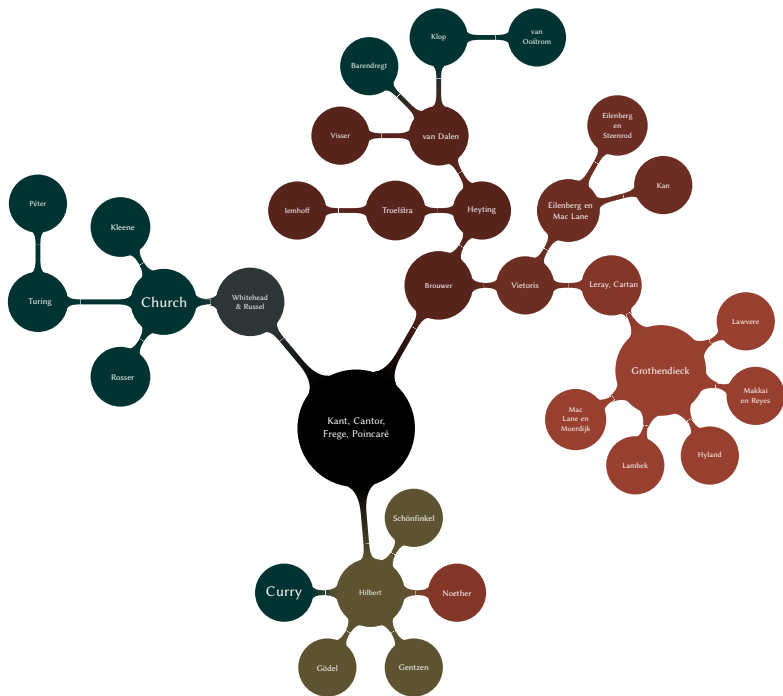


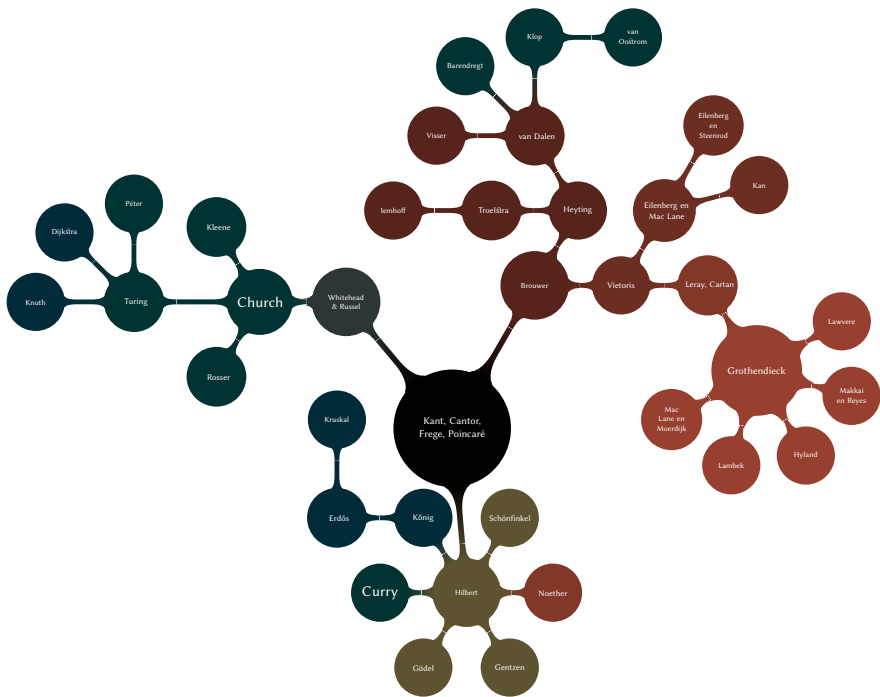


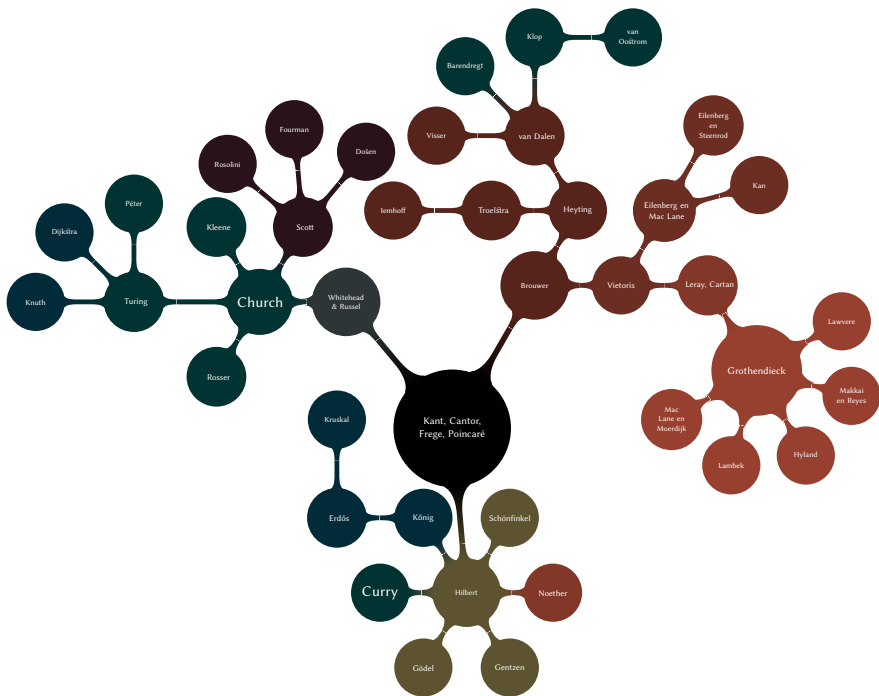


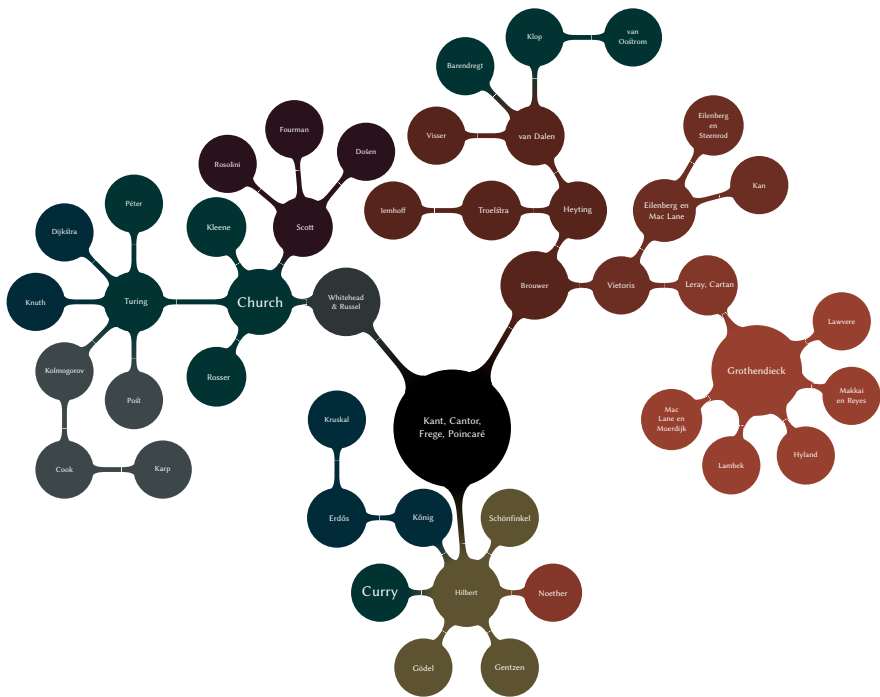


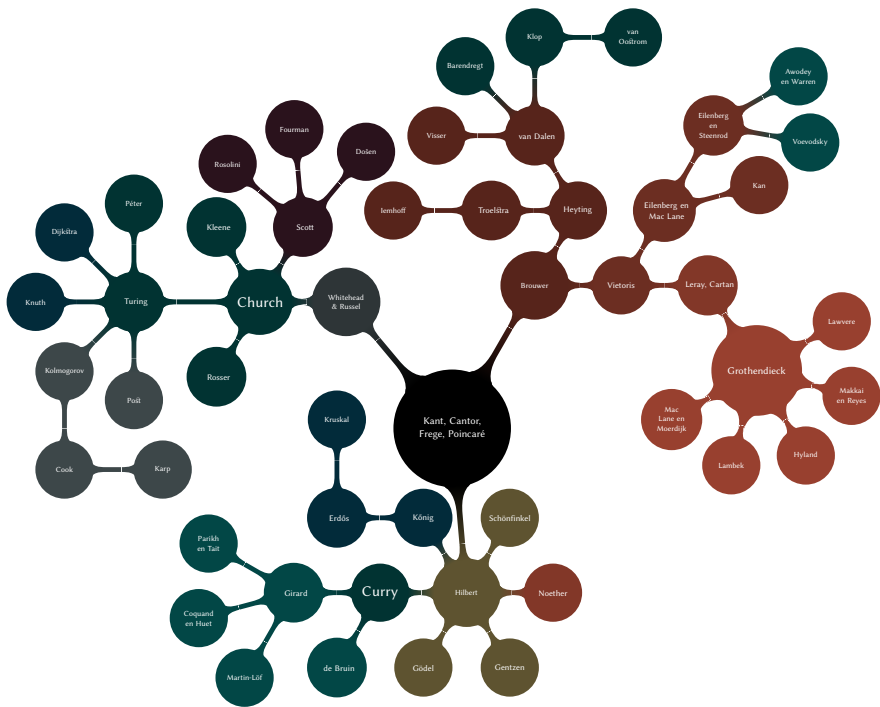






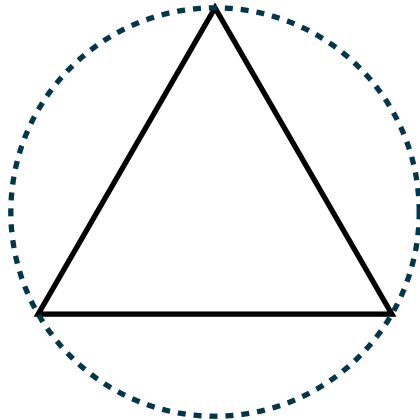




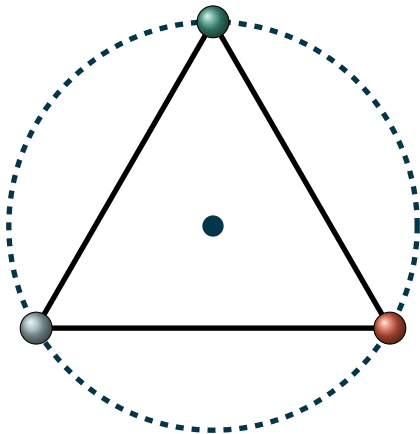


Herschrijven

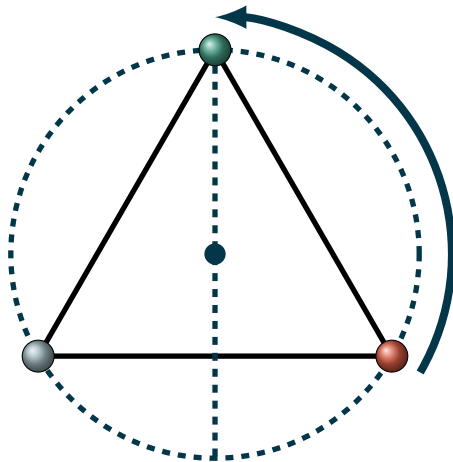
Dihedrale Groepen



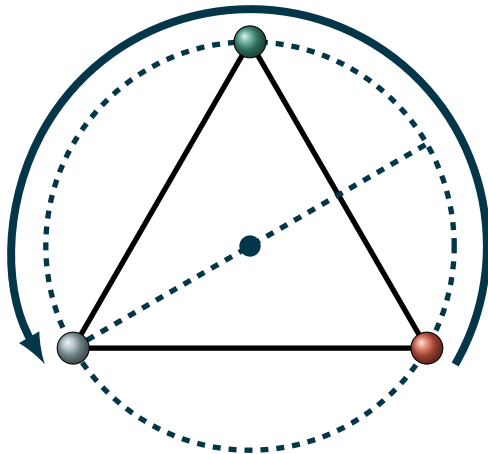
Dihedrale Groepen



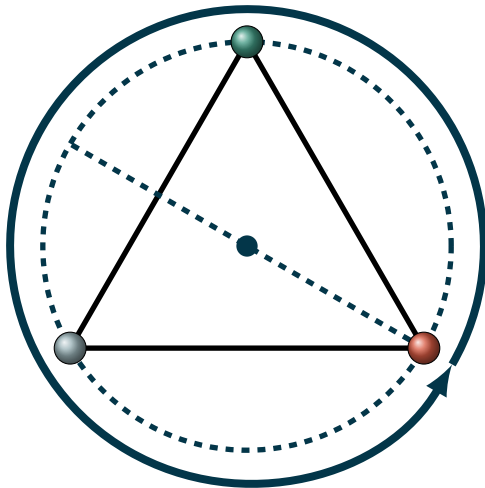
Dihedrale Groepen



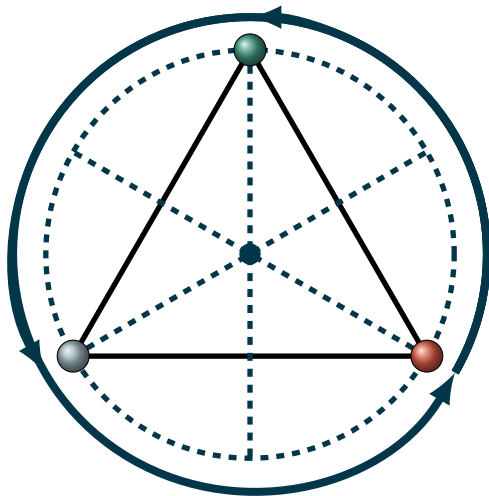
Dihedrale Groepen



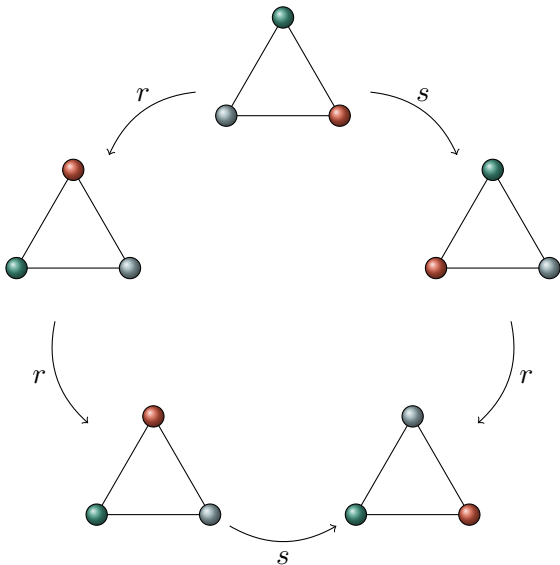
Dihedrale Groepen



Dihedrale Groepen



Herschrijven



Herschrijfregels

$$rrr \longrightarrow e$$

$$ss \longrightarrow e$$

$$sr \longrightarrow rrs$$

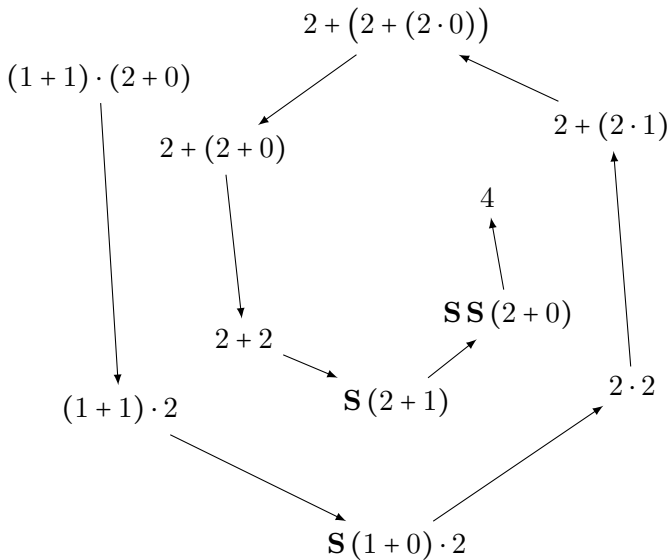
Rekenkunde

$$\begin{array}{l} n + 0 \longrightarrow n \\ n + (\mathbf{S} m) \longrightarrow \mathbf{S}(n + m) \end{array}$$

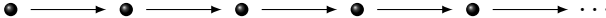
Rechenkunde

$$\begin{aligned}n + 0 &\longrightarrow n \\n + (\mathbf{S} m) &\longrightarrow \mathbf{S}(n + m) \\n \cdot 0 &\longrightarrow 0 \\n \cdot (\mathbf{S} m) &\longrightarrow n + (n \cdot m)\end{aligned}$$

Rekenen met Rekenkunde



Terminatie



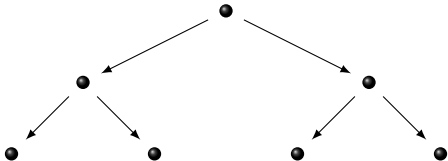
Terminatie



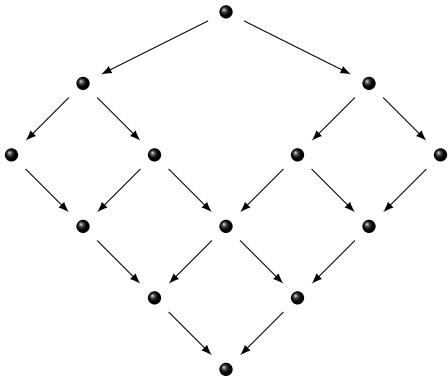
Amalie Emmy Noether



Confluent



Confluentie



Alonzo Church & Barkley Rosser



Uitbreidingen

$$n \cdot (x + y) \longrightarrow n \cdot x + n \cdot y$$

Uitbreidingen

$$n \cdot (x + y) \longrightarrow n \cdot x + n \cdot y \quad \checkmark$$

Uitbreidingen

$$\begin{array}{l} n \cdot (x + y) \longrightarrow n \cdot x + n \cdot y \quad \checkmark \\ 2 + 2 \longrightarrow 5 \end{array}$$

Uitbreidingen

$$\begin{array}{l} n \cdot (x + y) \longrightarrow n \cdot x + n \cdot y \quad \checkmark \\ 2 + 2 \longrightarrow 5 \quad \times \end{array}$$

Uitbreidingen

$$\begin{array}{rcll} n \cdot (x + y) & \longrightarrow & n \cdot x + n \cdot y & \checkmark \\ 2 + 2 & \longrightarrow & 5 & \times \\ 4 & \longrightarrow & 2 + 2 & \end{array}$$

Uitbreidingen

$$\begin{array}{rcll} n \cdot (x + y) & \longrightarrow & n \cdot x + n \cdot y & \checkmark \\ 2 + 2 & \longrightarrow & 5 & \times \\ 4 & \longrightarrow & 2 + 2 & \times \end{array}$$


Uitbreidingen

$$\begin{array}{rcll} n \cdot (x + y) & \longrightarrow & n \cdot x + n \cdot y & \checkmark \\ 2 + 2 & \longrightarrow & 5 & \times \\ 4 & \longrightarrow & 2 + 2 & \times \\ 2 + 2 & \longrightarrow & 4 & \end{array}$$

Uitbreidingen

$$\begin{array}{rcll} n \cdot (x + y) & \longrightarrow & n \cdot x + n \cdot y & \checkmark \\ 2 + 2 & \longrightarrow & 5 & \times \\ 4 & \longrightarrow & 2 + 2 & \times \\ 2 + 2 & \longrightarrow & 4 & \checkmark \end{array}$$

Kernbegrippen



Confluentie

Terminatie

Betekenis

Onbeslisbaarheid



„Recursive Unsolvability of a Problem of Thue” – Pošt (1947)

Typen

Implicationeel Fragment

$$\frac{}{\Gamma, \phi \vdash \phi} \text{AX}$$

$$\frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi \rightarrow \psi} \rightarrow\text{I}$$

$$\frac{\Gamma \vdash \phi \rightarrow \psi \quad \Gamma \vdash \phi}{\Gamma \vdash \psi} \rightarrow\text{E}$$

Bewijs van $\phi \rightarrow \phi$

Bewijs van $\phi \rightarrow \phi$

$$\frac{\overline{\phi \vdash \phi} \text{ AX}}{\vdash \phi \rightarrow \phi} \rightarrow\text{I}$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{}{\vdash \phi \rightarrow \psi \rightarrow \phi}$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{\overline{\phi \vdash \psi \rightarrow \phi}}{\vdash \phi \rightarrow \psi \rightarrow \phi} \rightarrow\text{I}$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{\frac{\frac{}{\psi, \phi \vdash \phi} \text{AX}}{\phi \vdash \psi \rightarrow \phi} \rightarrow\text{I}}{\vdash \phi \rightarrow \psi \rightarrow \phi} \rightarrow\text{I}}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\frac{\frac{\frac{\phi \rightarrow \psi \rightarrow \chi, \phi \rightarrow \psi, \phi \vdash \chi}{\phi \rightarrow \psi \rightarrow \chi, \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I}}{\phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\frac{\frac{\frac{\Gamma \vdash \chi}{\phi \rightarrow \psi \rightarrow \chi, \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I}}{\phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\frac{\frac{\frac{\frac{\Gamma \vdash \psi \rightarrow \chi}{\Gamma \vdash \chi} \rightarrow\text{I}}{\phi \rightarrow \psi \rightarrow \chi, \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I}}{\phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}}{\frac{\Gamma \vdash \psi}{\Gamma \vdash \psi} \rightarrow\text{E}} \rightarrow\text{E}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\Gamma \vdash \psi \rightarrow \chi}{\Gamma \vdash \psi \rightarrow \chi}}{\Gamma \vdash \chi}}{\phi \rightarrow \psi \rightarrow \chi, \phi \rightarrow \psi \vdash \phi \rightarrow \chi}}{\phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}}{\frac{\frac{\frac{\Gamma \vdash \phi \rightarrow \psi}{\Gamma \vdash \phi}}{\Gamma \vdash \psi}}{\Gamma \vdash \phi \rightarrow \psi}}{\Gamma \vdash \phi \rightarrow \psi}} \rightarrow E}{\Gamma \vdash \psi \rightarrow \chi} \rightarrow E} \rightarrow I$$

Bewijs van

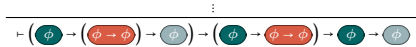
$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\frac{
 \frac{
 \frac{
 \frac{
 \frac{}{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi} \text{AX}
 }{\Gamma \vdash \psi \rightarrow \chi} \rightarrow\text{E}
 }{\Gamma \vdash \psi \rightarrow \chi} \text{AX}
 }{\Gamma \vdash \phi} \rightarrow\text{E}
 }{\Gamma \vdash \psi \rightarrow \chi} \text{AX}
 }{\Gamma \vdash \psi} \rightarrow\text{E}
 }{\Gamma \vdash \chi} \rightarrow\text{E}
 }{\Gamma \vdash \psi \rightarrow \chi} \rightarrow\text{E}
 }{\Gamma \vdash \psi \rightarrow \chi, \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I}
 }{\phi \rightarrow \psi \rightarrow \chi, \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I}
 }{\phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}
 }{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}$$

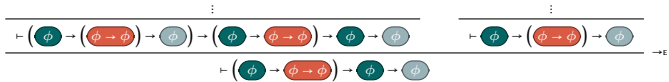
Alternatief bewijs voor $\phi \rightarrow \phi$

$$\vdash (\phi \rightarrow (\phi \rightarrow \phi)) \rightarrow \phi \rightarrow (\phi \rightarrow \phi \rightarrow \phi)$$

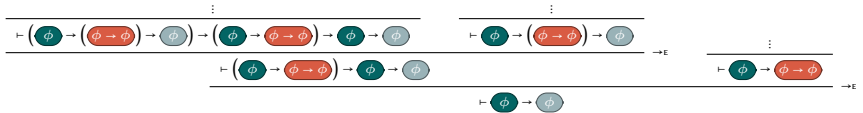
Alternatief bewijs voor $\phi \rightarrow \phi$



Alternatief bewijs voor $\phi \rightarrow \phi$



Alternatief bewijs voor $\phi \rightarrow \phi$



λ -Calculus

$$\frac{}{\Gamma, \phi \vdash \phi} \text{AX}$$

$$\frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi \rightarrow \psi} \rightarrow\text{I}$$

$$\frac{\Gamma \vdash \phi \rightarrow \psi \quad \Gamma \vdash \phi}{\Gamma \vdash \psi} \rightarrow\text{E}$$

λ -Calculus

$$\frac{}{\Gamma, \phi \vdash \phi} \text{AX}$$

$$\frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi \rightarrow \psi} \rightarrow\text{I}$$

$$\frac{\Gamma \vdash \phi \rightarrow \psi \quad \Gamma \vdash \phi}{\Gamma \vdash \psi} \rightarrow\text{E}$$

λ -Calculus

$$\frac{}{\Gamma, \boxed{x} : \phi \vdash \boxed{x} : \phi} \text{AX}$$

$$\frac{\Gamma, \quad \phi \vdash \quad \psi}{\Gamma \vdash \quad \phi \rightarrow \psi} \rightarrow\text{I}$$

$$\frac{\Gamma \vdash \quad \phi \rightarrow \psi \quad \Gamma \vdash \quad \phi}{\Gamma \vdash \quad \psi} \rightarrow\text{E}$$

λ -Calculus

$$\frac{}{\Gamma, \boxed{x} : \phi \vdash \boxed{x} : \phi} \text{AX}$$

$$\frac{\Gamma, \boxed{x} : \phi \vdash \boxed{M} : \psi}{\Gamma \vdash \boxed{\lambda x^\phi. M} : \phi \rightarrow \psi} \rightarrow\text{I}$$

$$\frac{\Gamma \vdash \quad \phi \rightarrow \psi \quad \Gamma \vdash \quad \phi}{\Gamma \vdash \quad \psi} \rightarrow\text{E}$$

λ -Calculus

$$\frac{}{\Gamma, \boxed{x} : \phi \vdash \boxed{x} : \phi} \text{AX}$$

$$\frac{\Gamma, \boxed{x} : \phi \vdash \boxed{M} : \psi}{\Gamma \vdash \boxed{\lambda x^\phi. M} : \phi \rightarrow \psi} \rightarrow\text{I}$$

$$\frac{\Gamma \vdash \boxed{M} : \phi \rightarrow \psi \quad \Gamma \vdash \boxed{N} : \phi}{\Gamma \vdash \boxed{MN} : \psi} \rightarrow\text{E}$$

Bewijs van $\phi \rightarrow \phi$

$$\frac{\frac{}{\phi \vdash \phi} \text{AX}}{\vdash \phi \rightarrow \phi} \rightarrow\text{I}$$

Bewijs van $\phi \rightarrow \phi$

$$\frac{\frac{}{x : \phi \vdash \phi} \text{AX}}{\vdash \phi \rightarrow \phi} \rightarrow\text{I}$$

Bewijs van $\phi \rightarrow \phi$

$$\frac{\frac{}{\vdash x : \phi} \text{AX} \quad x : \phi \vdash x : \phi}{\vdash \phi \rightarrow \phi} \rightarrow\text{I}}$$

Bewijs van $\phi \rightarrow \phi$

$$\frac{\frac{}{x : \phi \vdash x : \phi} \text{AX}}{\vdash \lambda x^\phi. x : \phi \rightarrow \phi} \rightarrow\text{I}$$

Identiteitsfunctie

I. Identiteitsfunctie.

$$Ix = x \text{ for all } x.$$

or in my notation

$$I @ x = x.$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{\frac{\frac{}{\psi, \phi \vdash \phi} \text{AX}}{} \rightarrow\text{I}}{\phi \vdash \psi \rightarrow \phi} \rightarrow\text{I}}{\vdash \phi \rightarrow \psi \rightarrow \phi} \rightarrow\text{I}$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{\frac{\frac{}{y : \psi, x : \phi \vdash \phi} \text{AX}}{} \rightarrow\text{I}}{x : \phi \vdash \psi \rightarrow \phi} \rightarrow\text{I}}{\vdash \phi \rightarrow \psi \rightarrow \phi} \rightarrow\text{I}$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{\frac{\frac{}{y : \psi, x : \phi \vdash x : \phi} \text{AX}}{\vdash x : \phi} \rightarrow\text{I}}{\vdash \psi \rightarrow \phi} \rightarrow\text{I}}{\vdash \phi \rightarrow \psi \rightarrow \phi} \rightarrow\text{I}$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{\frac{\frac{}{y : \psi, x : \phi \vdash x : \phi} \text{AX}}{} \rightarrow \text{I}}{x : \phi \vdash \lambda y^\psi. x : \psi \rightarrow \phi} \rightarrow \text{I}}{\vdash \phi \rightarrow \psi \rightarrow \phi} \rightarrow \text{I}$$

Bewijs van $\phi \rightarrow \psi \rightarrow \phi$

$$\frac{\frac{\frac{}{y : \psi, x : \phi \vdash x : \phi} \text{AX}}{\vdash x : \phi \vdash \lambda y^\psi . x : \psi \rightarrow \phi} \rightarrow\text{I}}{\vdash \lambda x^\phi . \lambda y^\psi . x : \phi \rightarrow \psi \rightarrow \phi} \rightarrow\text{I}}{\vdash \lambda x^\phi . \lambda y^\psi . x : \phi \rightarrow \psi \rightarrow \phi} \rightarrow\text{I}$$

Constante functie-functie

C. Constante functie.

$$Cxy = x \quad \text{for all } x, y.$$

$$\text{ü. } C @ x @ y = x \quad \text{for all } x \text{ r } y$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := \quad \phi, \quad \phi \rightarrow \psi, \quad \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
 \frac{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi \quad \text{AX}}{\Gamma \vdash \psi \rightarrow \chi} \rightarrow\text{E} \quad \frac{\Gamma \vdash \phi \quad \text{AX}}{\Gamma \vdash \psi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \psi \rightarrow \chi \quad \Gamma \vdash \psi}{\Gamma \vdash \chi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \chi}{\phi \rightarrow \psi \rightarrow \chi, \quad \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{\phi \rightarrow \psi \rightarrow \chi, \quad \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{\phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{\phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$

$$\begin{array}{c}
 \frac{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi \text{ AX}}{\Gamma \vdash \psi \rightarrow \chi} \rightarrow E \quad \frac{\Gamma \vdash \phi \text{ AX}}{\Gamma \vdash \psi} \rightarrow E \\
 \frac{\Gamma \vdash \psi \rightarrow \chi \quad \Gamma \vdash \psi}{\Gamma \vdash \chi} \rightarrow E \\
 \frac{\Gamma \vdash \chi}{f : \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow I \\
 \frac{f : \phi \rightarrow \psi \rightarrow \chi \vdash \quad \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{(f : \phi \rightarrow \psi \rightarrow \chi) \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I \\
 \frac{}{\vdash (f : \phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$

$$\begin{array}{c}
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \quad \text{AX}}{\Gamma \vdash \psi \rightarrow \chi} \rightarrow\text{E} \quad \frac{\Gamma \vdash \phi \quad \text{AX}}{\Gamma \vdash \psi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \psi \rightarrow \chi \quad \Gamma \vdash \psi}{\Gamma \vdash \chi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \chi}{f : \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{f : \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
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 \frac{\Gamma \vdash \psi \rightarrow \chi \quad \Gamma \vdash \psi}{\Gamma \vdash \chi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \chi}{f : \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{f : \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

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$$\begin{array}{c}
 \frac{\frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi}{\Gamma \vdash \psi \rightarrow \chi} \text{AX} \quad \frac{\Gamma \vdash x : \phi}{\Gamma \vdash \chi} \text{AX}}{\Gamma \vdash \psi \rightarrow \chi} \rightarrow\text{E} \quad \frac{\frac{\Gamma \vdash g : \phi \rightarrow \psi}{\Gamma \vdash \psi} \text{AX} \quad \frac{\Gamma \vdash x : \phi}{\Gamma \vdash \chi} \text{AX}}{\Gamma \vdash \psi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \psi \rightarrow \chi \quad \Gamma \vdash \psi}{\Gamma \vdash \chi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \chi}{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I}
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := \mathbf{x} : \phi, \mathbf{g} : \phi \rightarrow \psi, \mathbf{f} : \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\Gamma \vdash \mathbf{f} : \phi \rightarrow \psi \rightarrow \chi}{\text{AX}}}{\text{AX}}}{\text{AX}}}{\Gamma \vdash \mathbf{f} \mathbf{x} : \psi \rightarrow \chi} \quad \frac{\frac{\frac{\Gamma \vdash \mathbf{x} : \phi}{\text{AX}}}{\text{AX}}}{\Gamma \vdash \psi} \rightarrow\text{E} \quad \frac{\frac{\frac{\Gamma \vdash \mathbf{g} : \phi \rightarrow \psi}{\text{AX}}}{\text{AX}}}{\Gamma \vdash \psi} \rightarrow\text{E} \quad \frac{\frac{\frac{\Gamma \vdash \mathbf{x} : \phi}{\text{AX}}}{\text{AX}}}{\Gamma \vdash \chi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \mathbf{f} \mathbf{x} : \psi \rightarrow \chi \quad \Gamma \vdash \psi}{\Gamma \vdash \chi} \rightarrow\text{E} \\
 \frac{\Gamma \vdash \chi}{\Gamma \vdash \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{\Gamma \vdash \phi \rightarrow \chi, \Gamma \vdash \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{\Gamma \vdash \phi \rightarrow \psi \rightarrow \chi \vdash \phi \rightarrow \psi \rightarrow \chi}{\vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow\text{I} \\
 \frac{\vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi} \rightarrow\text{I}
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
 \frac{\frac{\frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi}{\text{AX}} \quad \frac{\Gamma \vdash x : \phi}{\text{AX}}}{\Gamma \vdash f x : \psi \rightarrow \chi} \rightarrow E \quad \frac{\frac{\frac{\Gamma \vdash g : \phi \rightarrow \psi}{\text{AX}} \quad \frac{\Gamma \vdash x : \phi}{\text{AX}}}{\Gamma \vdash g x : \psi} \rightarrow E}{\Gamma \vdash \chi} \rightarrow E}{\Gamma \vdash \chi} \rightarrow I \\
 \frac{\frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi, \Gamma \vdash g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \quad \text{AX}}{\Gamma \vdash f x : \psi \rightarrow \chi} \quad \frac{\Gamma \vdash x : \phi \quad \text{AX}}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \quad \frac{\Gamma \vdash g : \phi \rightarrow \psi \quad \text{AX}}{\Gamma \vdash g x : \psi} \quad \frac{\Gamma \vdash x : \phi \quad \text{AX}}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \\
 \frac{\Gamma \vdash f x : \psi \rightarrow \chi \quad \Gamma \vdash g x : \psi}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \\
 \frac{\Gamma \vdash (f x) (g x) : \chi}{f : \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi} \rightarrow I \\
 \frac{f : \phi \rightarrow \psi \rightarrow \chi, g : \phi \rightarrow \psi \vdash \phi \rightarrow \chi}{f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I \\
 \frac{f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \quad \text{AX}}{\Gamma \vdash f x : \psi \rightarrow \chi} \quad \frac{\Gamma \vdash x : \phi \quad \text{AX}}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \\
 \frac{\Gamma \vdash g : \phi \rightarrow \psi \quad \text{AX} \quad \Gamma \vdash x : \phi \quad \text{AX}}{\Gamma \vdash g x : \psi} \rightarrow E \\
 \frac{\Gamma \vdash (f x) (g x) : \chi}{\Gamma \vdash \lambda x^\phi. (f x) (g x) : \phi \rightarrow \chi} \rightarrow I \\
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi, \Gamma \vdash g : \phi \rightarrow \psi \vdash \lambda x^\phi. (f x) (g x) : \phi \rightarrow \chi}{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I \\
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \vdash (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \quad \text{AX}}{\Gamma \vdash f x : \psi \rightarrow \chi} \quad \frac{\Gamma \vdash x : \phi \quad \text{AX}}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \quad \frac{\Gamma \vdash g : \phi \rightarrow \psi \quad \text{AX}}{\Gamma \vdash g x : \psi} \quad \frac{\Gamma \vdash x : \phi \quad \text{AX}}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \\
 \frac{\Gamma \vdash f x : \psi \rightarrow \chi \quad \Gamma \vdash g x : \psi}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \\
 \frac{\Gamma \vdash (f x) (g x) : \chi}{\Gamma \vdash \lambda x^\phi. (f x) (g x) : \phi \rightarrow \chi} \rightarrow I \\
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi, \Gamma \vdash g : \phi \rightarrow \psi \vdash \lambda x^\phi. (f x) (g x) : \phi \rightarrow \chi}{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \vdash \lambda g^{\phi \rightarrow \psi}. \lambda x^\phi. (f x) (g x) : (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I \\
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \vdash \lambda g^{\phi \rightarrow \psi}. \lambda x^\phi. (f x) (g x) : (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I
 \end{array}$$

Bewijs van

$$(\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi$$

$$\Gamma := x : \phi, g : \phi \rightarrow \psi, f : \phi \rightarrow \psi \rightarrow \chi$$

$$\begin{array}{c}
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi}{\Gamma \vdash f x : \psi \rightarrow \chi} \text{AX} \quad \frac{\Gamma \vdash x : \phi}{\Gamma \vdash (f x) (g x) : \chi} \text{AX} \quad \frac{\Gamma \vdash g : \phi \rightarrow \psi}{\Gamma \vdash g x : \psi} \text{AX} \quad \frac{\Gamma \vdash x : \phi}{\Gamma \vdash (f x) (g x) : \chi} \text{AX} \\
 \frac{\Gamma \vdash f x : \psi \rightarrow \chi \quad \Gamma \vdash g x : \psi}{\Gamma \vdash (f x) (g x) : \chi} \rightarrow E \\
 \frac{\Gamma \vdash (f x) (g x) : \chi}{\Gamma \vdash \lambda x^\phi. (f x) (g x) : \phi \rightarrow \chi} \rightarrow I \\
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi, \Gamma \vdash g : \phi \rightarrow \psi \vdash \lambda x^\phi. (f x) (g x) : \phi \rightarrow \chi}{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \vdash \lambda g^{\phi \rightarrow \psi}. \lambda x^\phi. (f x) (g x) : (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi} \rightarrow I \\
 \frac{\Gamma \vdash f : \phi \rightarrow \psi \rightarrow \chi \vdash \lambda g^{\phi \rightarrow \psi}. \lambda x^\phi. (f x) (g x) : (\phi \rightarrow \psi) \rightarrow \phi \rightarrow \chi}{\vdash \lambda f^{\phi \rightarrow \psi \rightarrow \chi}. \lambda g^{\phi \rightarrow \psi}. \lambda x^\phi. (f x) (g x) : (\phi \rightarrow \psi \rightarrow \chi) \rightarrow (\phi \rightarrow \psi) \rightarrow \phi} \rightarrow I
 \end{array}$$

Fusiefunctie

S: Verschmelzungsfunktion

$$S \phi \otimes x = (\phi \otimes x)$$

$$- S \phi \otimes x \otimes x = \phi \otimes x \otimes (x \otimes x)$$

Omwegen

$$\frac{\frac{\frac{}{\phi \vdash} \text{AX}}{\vdash} \quad \phi \rightarrow \phi}{\vdash} \rightarrow\text{I}}{\phi \vdash} \quad \frac{\frac{}{\phi \vdash} \text{AX}}{\phi} \rightarrow\text{E}}$$

Omwegen

$$\frac{\frac{\frac{}{\phi \vdash} \text{AX}}{\phi \vdash} \rightarrow\text{I}}{\vdash \quad \phi \rightarrow \phi} \rightarrow\text{I} \quad \frac{}{\phi \vdash \quad \phi} \text{AX}}{\phi \vdash \quad \phi} \rightarrow\text{E}$$

$$\frac{}{\phi \vdash \quad \phi} \text{AX}$$

Omwegen

$$\frac{\frac{\frac{}{x : \phi \vdash x : \phi} \text{AX}}{\vdash \lambda x^\phi. x : \phi \rightarrow \phi} \rightarrow I}{\frac{}{y : \phi \vdash y : \phi} \text{AX}}{\frac{}{y : \phi \vdash (\lambda x^\phi. x) y : \phi} \rightarrow E} \rightarrow E}$$

$$\frac{}{y : \phi \vdash x[x \mapsto y] : \phi} \text{AX}$$

Omwegen

$$\frac{\frac{\vdots}{\Gamma, x : \phi \vdash M : \psi}}{\Gamma \vdash \lambda x^\phi. M : \phi \rightarrow \psi} \rightarrow I \quad \frac{\Gamma \vdash y : \phi \vdash y : \phi}{\Gamma, y : \phi \vdash (\lambda x^\phi. M) y : \psi} \rightarrow E \quad \text{AX}$$

$$\frac{\vdots}{\Gamma, y : \phi \vdash M[x \mapsto y] : \psi}$$

Omwegen

$$\frac{\frac{\frac{\vdots}{\Gamma, x : \phi \vdash M : \psi}}{\Gamma \vdash \lambda x^\phi. M : \phi \rightarrow \psi} \rightarrow_I \quad \frac{\frac{\vdots}{\Delta \vdash N : \phi}}{\Gamma, \Delta \vdash (\lambda x^\phi. M) N : \psi} \rightarrow_E}{\Gamma, \Delta \vdash (\lambda x^\phi. M) N : \psi}$$

$$\frac{\frac{\vdots}{\Gamma, \Delta \vdash M[x \mapsto N] : \psi}}$$


Evaluatie

$$(\lambda x^\phi. M) N = M[x \mapsto N]$$

Evaluatie

$$(\lambda x^\phi. M) N \longrightarrow M[x \mapsto N]$$

Kernbegrippen



Confluentie

Terminatie

Betekenis

Bewijzen voor $\phi \rightarrow \phi$

$$S := \lambda f^{A \rightarrow B \rightarrow A}. \lambda g^{A \rightarrow B}. \lambda x^A. f x (g x), \quad K := \lambda p^P. \lambda q^Q. p$$

SKK

Bewijzen voor $\phi \rightarrow \phi$

$$K := \lambda p^P . \lambda q^Q . p$$

$$(\lambda f^{A \rightarrow B \rightarrow A} . \lambda g^{A \rightarrow B} . \lambda x^A . f x (g x)) K K$$

Bewijzen voor $\phi \rightarrow \phi$

$$K := \lambda p^P . \lambda q^Q . p$$

$$(\lambda f^{\phi \rightarrow (\phi \rightarrow \phi) \rightarrow \phi} . \lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^\phi . f x (g x)) K K$$

Bewijzen voor $\phi \rightarrow \phi$

$$K := \lambda p^P. \lambda q^Q. p$$

$$(\lambda f^{\phi \rightarrow (\phi \rightarrow \phi)} \rightarrow \phi. \lambda g^{\phi \rightarrow (\phi \rightarrow \phi)}. \lambda x^\phi. f x (g x)) K K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)}. \lambda x^\phi. K x (g x)) K$$

Bewijzen voor $\phi \rightarrow \phi$

$$(\lambda f^{\phi \rightarrow (\phi \rightarrow \phi) \rightarrow \phi} . \lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . f x (g x)) K K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda p^P . \lambda q^Q . p) x (g x)) K$$

Bewijzen voor $\phi \rightarrow \phi$

$$(\lambda f^{\phi \rightarrow (\phi \rightarrow \phi)} \rightarrow \phi . \lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . f x (g x)) K K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda p^{\phi} . \lambda q^{\phi \rightarrow \phi} . p) x (g x)) K$$

Bewijzen voor $\phi \rightarrow \phi$

$$(\lambda f^{\phi \rightarrow (\phi \rightarrow \phi) \rightarrow \phi} . \lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . f x (g x)) K K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda p^{\phi} . \lambda q^{\phi \rightarrow \phi} . p) x (g x)) K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda q^{\phi \rightarrow \phi} . x) (g x)) K$$

Bewijzen voor $\phi \rightarrow \phi$

$$(\lambda f^{\phi \rightarrow (\phi \rightarrow \phi)} \rightarrow \phi . \lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . f x (g x)) K K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda p^{\phi} . \lambda q^{\phi \rightarrow \phi} . p) x (g x)) K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda q^{\phi \rightarrow \phi} . x) (g x)) K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . x) K$$

Bewijzen voor $\phi \rightarrow \phi$

$$(\lambda f^{\phi \rightarrow (\phi \rightarrow \phi)} \rightarrow \phi . \lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . f x (g x)) K K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda p^{\phi} . \lambda q^{\phi \rightarrow \phi} . p) x (g x)) K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . (\lambda q^{\phi \rightarrow \phi} . x) (g x)) K$$

↓

$$(\lambda g^{\phi \rightarrow (\phi \rightarrow \phi)} . \lambda x^{\phi} . x) K$$

↓

$$\lambda x^{\phi} . x$$

Gelijkheid

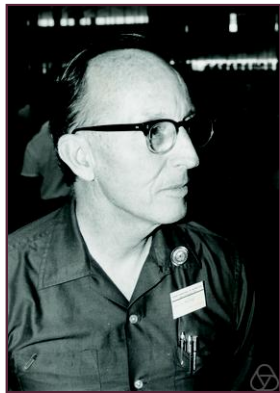
These 5 functions are not independent but.

$$I = SCC : S @ C @ C = S @ C @ x \text{ for any } x$$

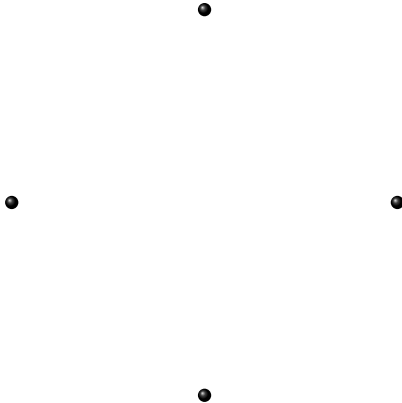
Curry-Howard correspondentie

type	formule
λ -term	bewijs
redex	omweg
bewoning	bewijsbaarheid

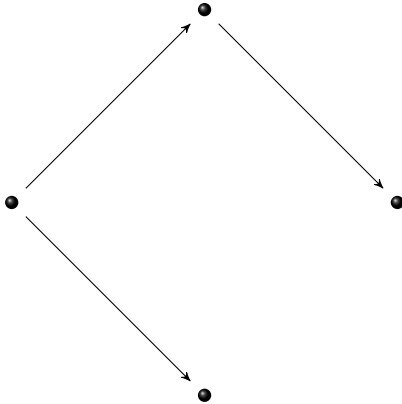
Samuel Eilenberg & Saunders Mac Lane



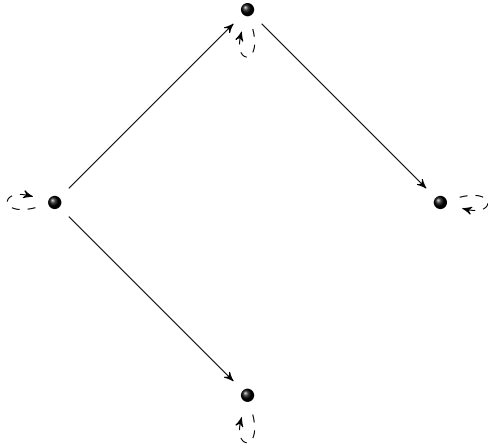
Categoriëen



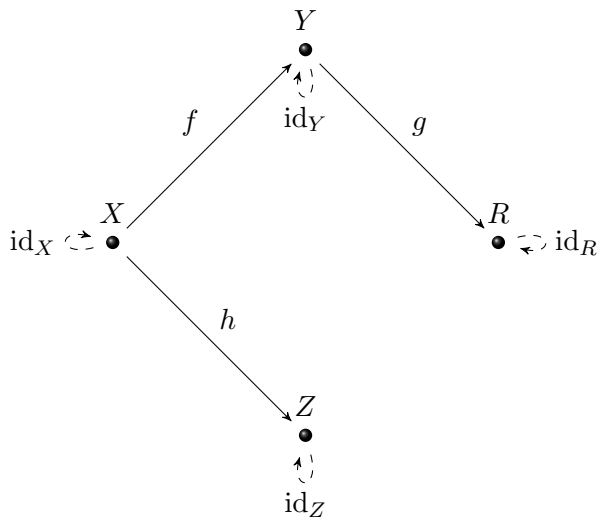
Categoriëen



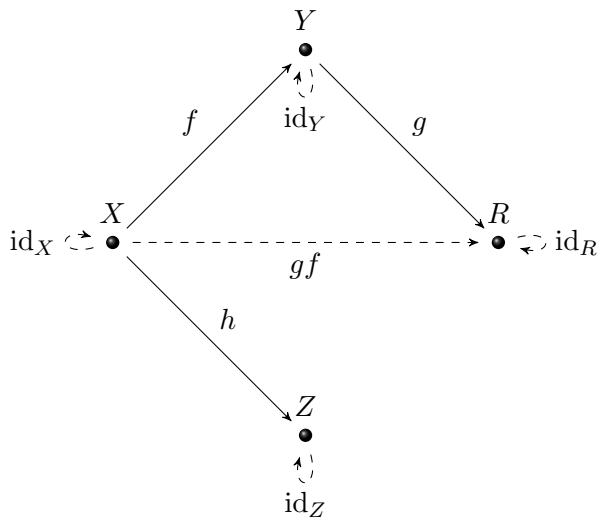
Categoriën



Categoriën



Categorien



Verzamelingen

Verzamelingen & Functies

Verzamelingen

Verzamelingen & Bijecties

Verzamelingen

Verzamelingen & Relaties

Syntactische Kategorie

Curry-Howard correspondentie

type	formule
λ -term	bewijs
redex	omweg
bewoning	bewijsbaarheid

Curry-Howard correspondentie

type
 λ -term
redex
bewoning

formule
bewijs
omweg
bewijsbaarheid

object
pijl
gelijkheid
verbondenheid

Toepassing

Bewijsassistenten

Coq

Bewijsassistenten

Coq
Inductive

Bewijsassistenten

Coq
Inductive
Calculus of Constructions

Bewijsassistenten

Coq
Inductive
Calculus of Constructions
Demo

Coq identiteisbewijs

```
Lemma I:A -> A.  
intro hypothese.  
apply hypothese.  
Qed.
```

Afbeeldingen

- ▶ Alonzo Church: University of St. Andrews
- ▶ Barkley Rosser: Society for Industrial and Applied Mathematics
- ▶ Emmy Noether: Physikerinnen.de
- ▶ Emil Leon Post: University of St. Andrews
- ▶ Samuel Eilenberg: Oberwolfach Photo Collection
- ▶ Saunders Mac Lane: Oberwolfach Photo Collection
- ▶ Schönfinkel's combinatoren, University of Lethbridge, Works of Haskell Curry collection