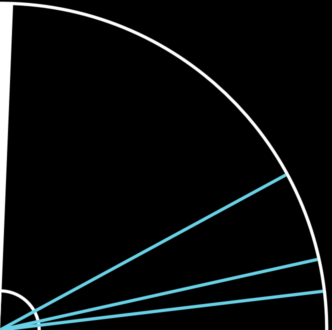
Admissibility & Unification

Jeroen Goudsmit
Utrecht University
ALCOP, May 15th 2014

Overview

Overview



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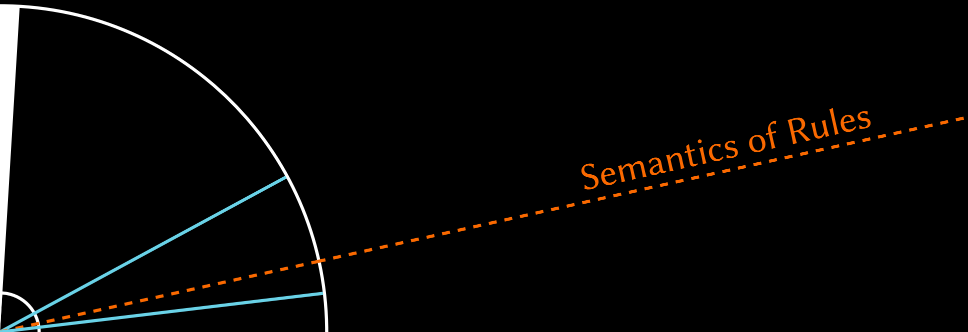
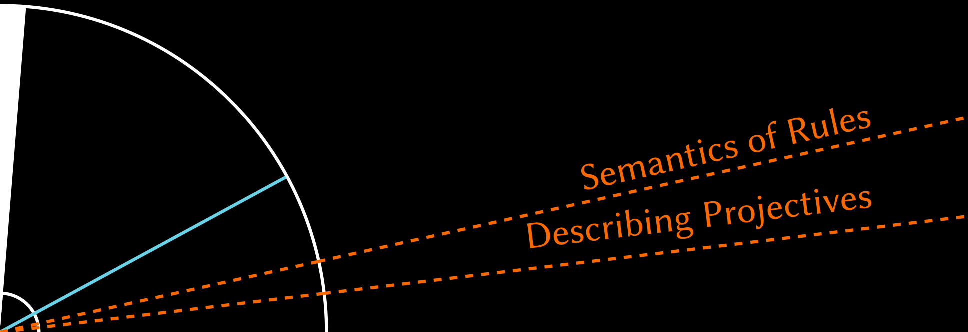
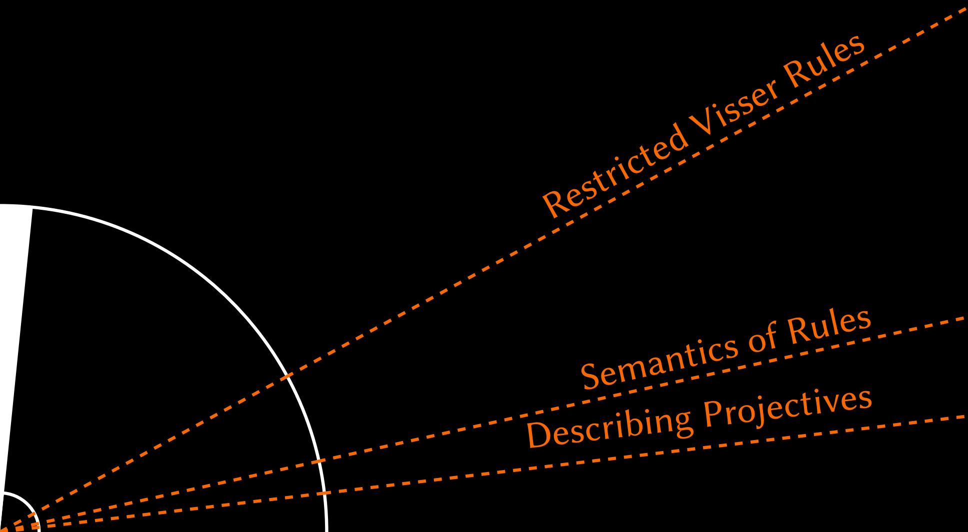


Overview



Overview


Restricted Visser Rules
Semantics of Rules
Describing Projectives

A / $\triangle$ admissible

σA is derivable

$$A \;/\; \Delta \text{ admissible}$$

σC is derivable for some $C \in \Delta$

σA is derivable

$A \mathrel{|\!\sim} \Delta$ admissible

σC is derivable for some $C \in \Delta$

Disjunction Property

$$\frac{A \vee B \text{ derivable}}{A \text{ derivable or } B \text{ derivable}}$$

Disjunction Property

$$\frac{p \vee q}{\{\ p, q\ \}}$$

$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$

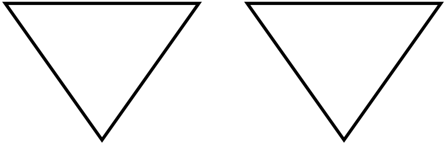
syntax

$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$

semantics

syntax

$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$



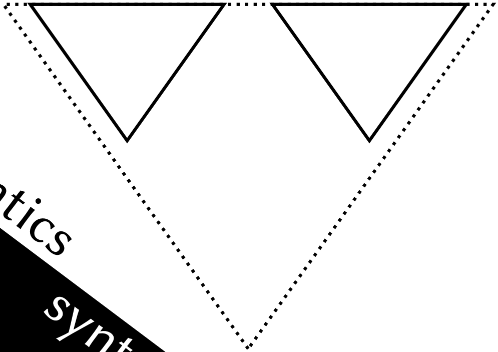
semantics

syntax

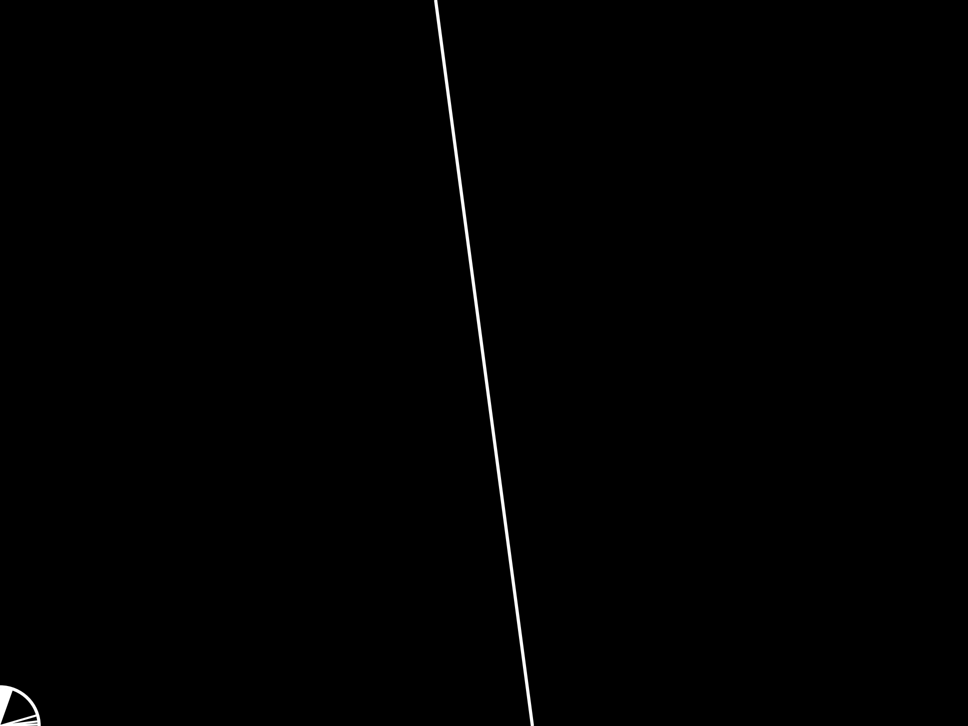
$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$

semantics

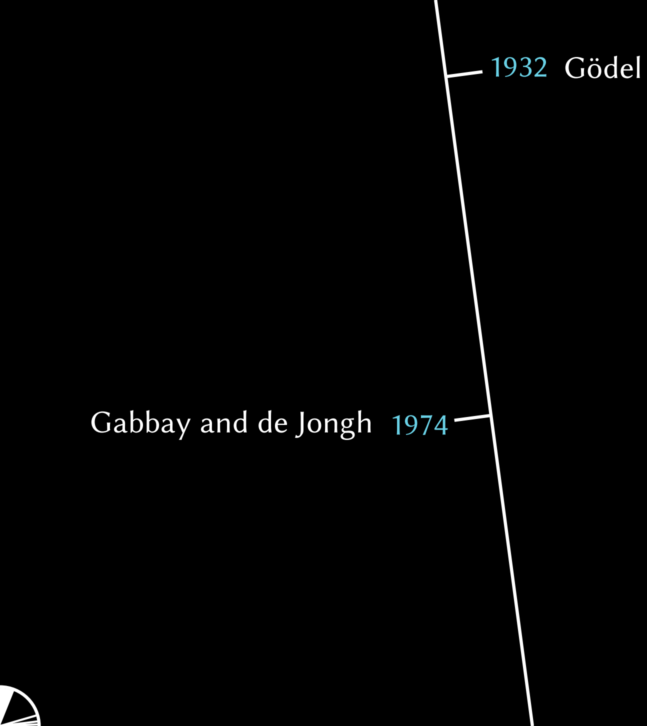
syntax

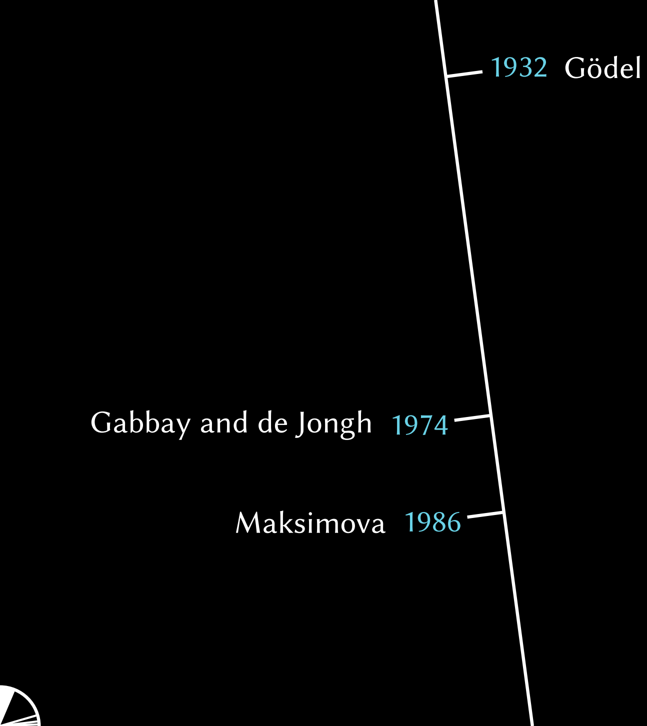


$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$



1932 Gödel


1932 Gödel
Gabbay and de Jongh 1974


1932 Gödel
Gabbay and de Jongh 1974
Maksimova 1986

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Gabbay and de Jongh 1974

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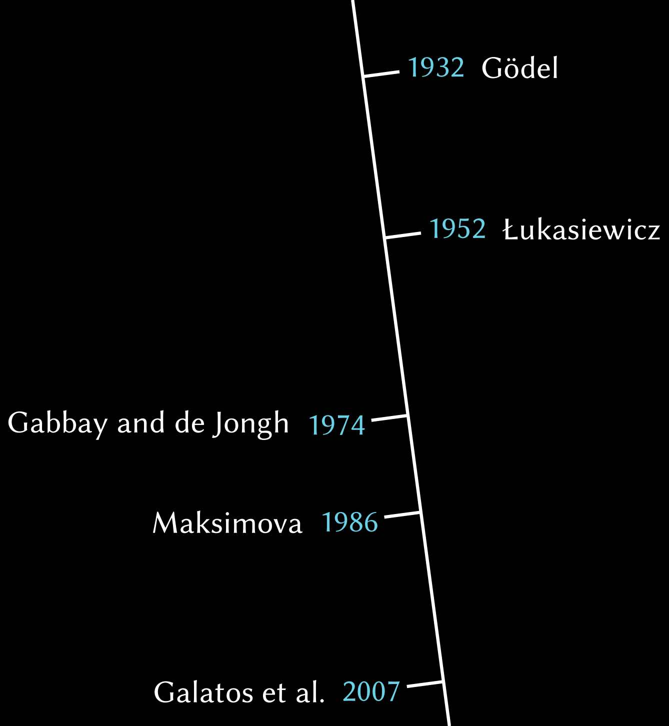
Galatos et al. 2007



1932 Gödel
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Gabbay and de Jongh 1974
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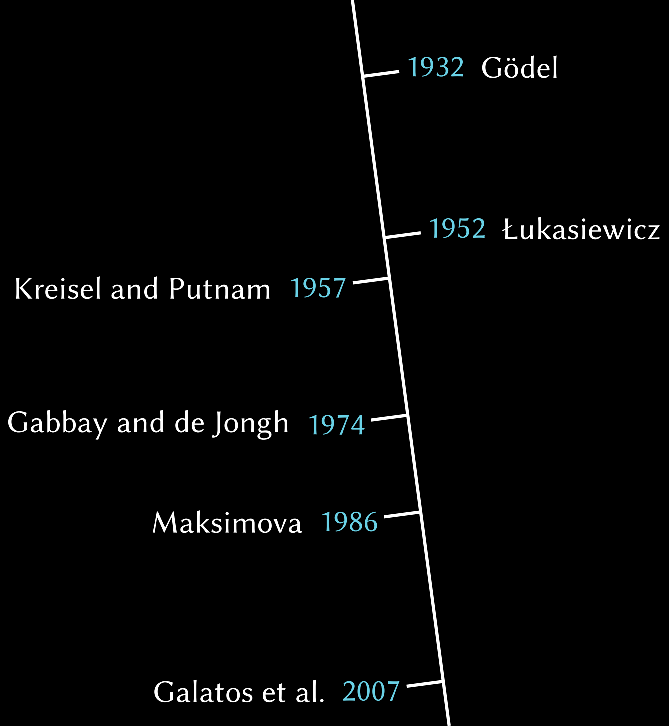
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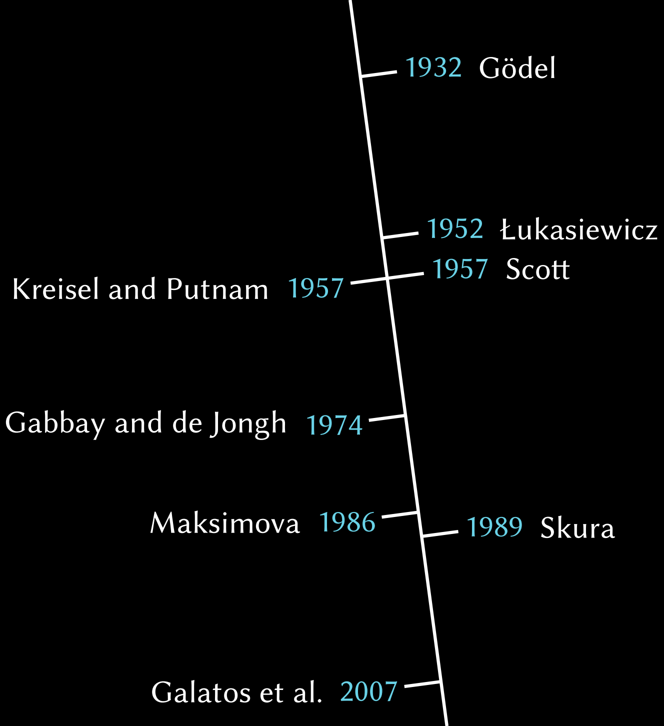
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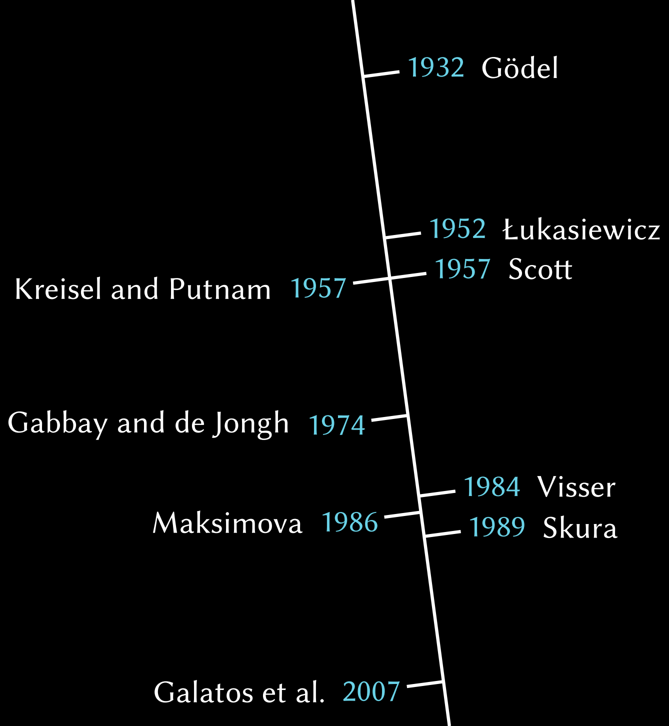
1989 Skura
Galatos et al. 2007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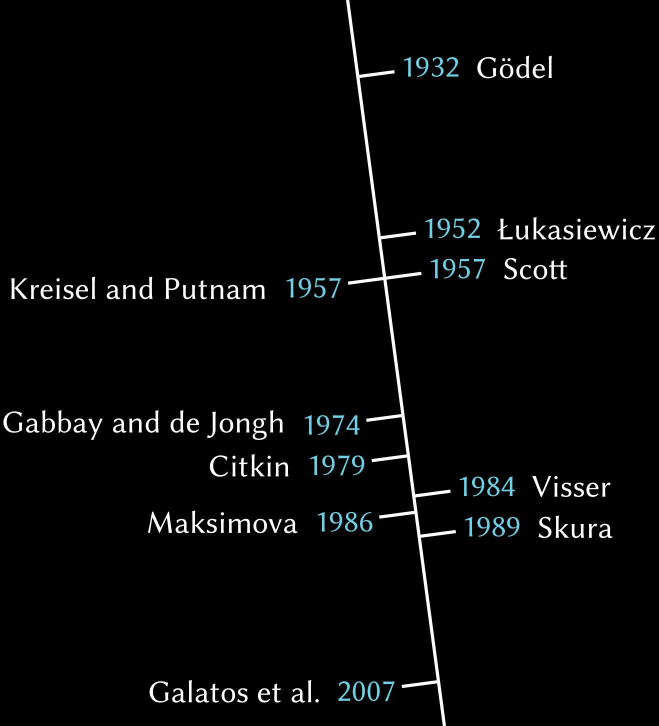
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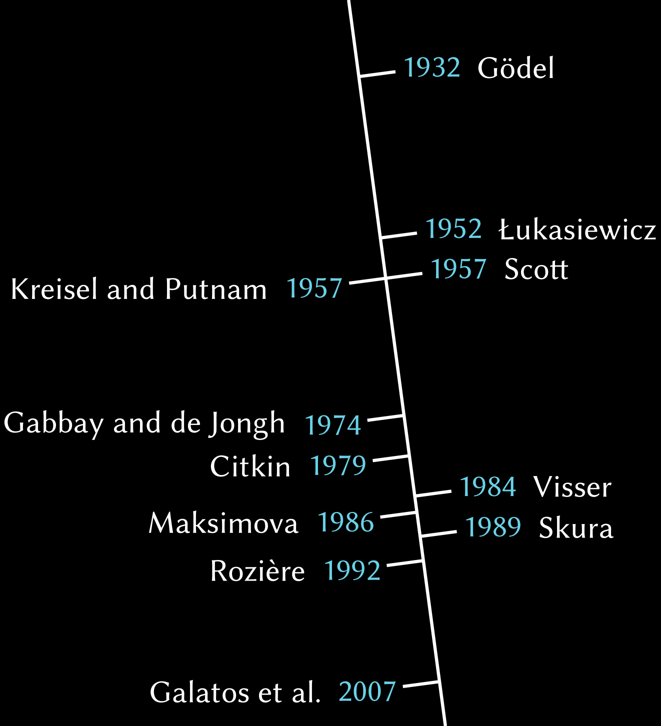
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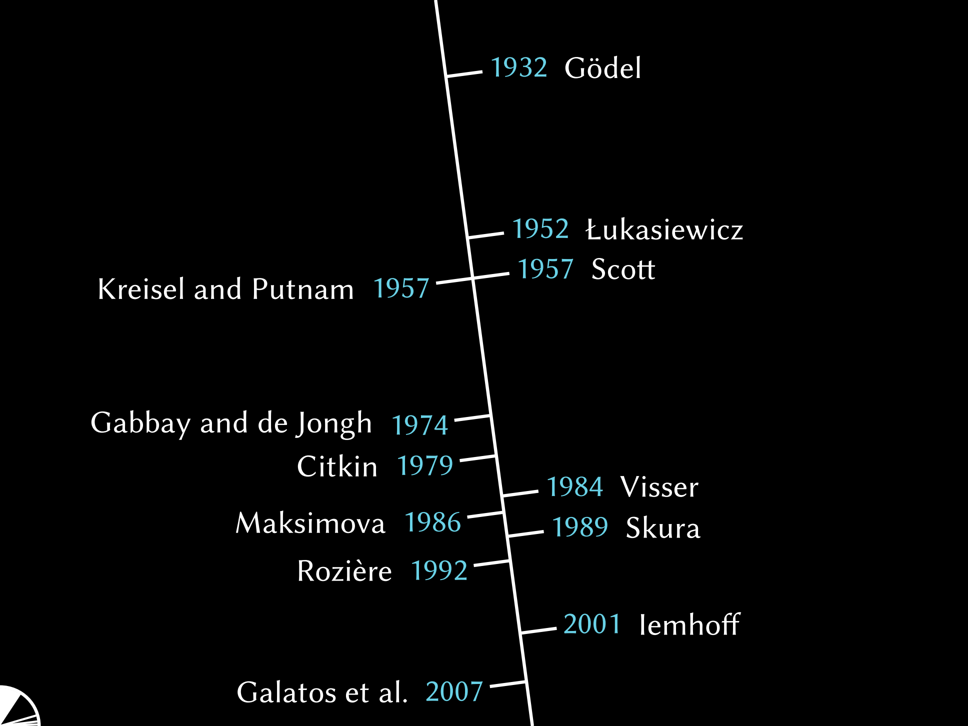
Galatos et al. 2007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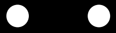
Analogous



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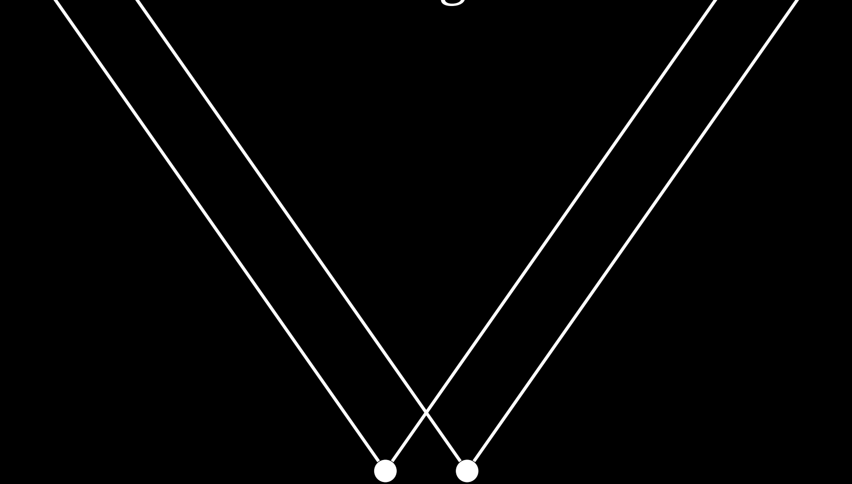

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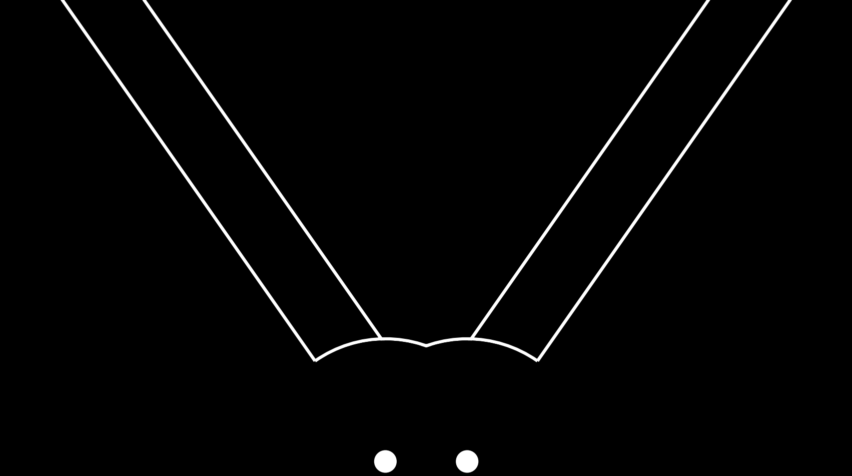

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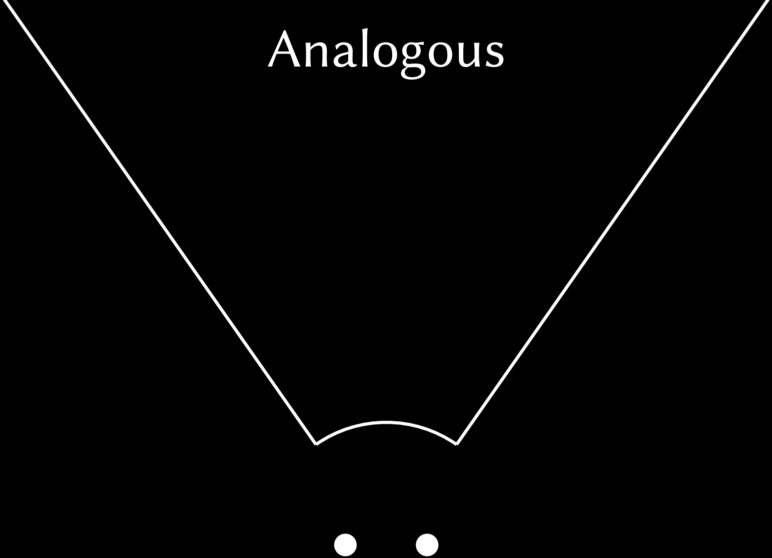

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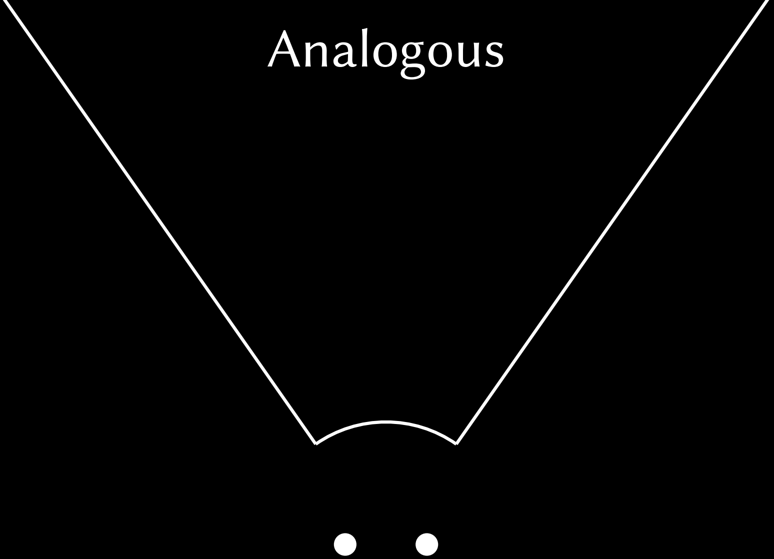
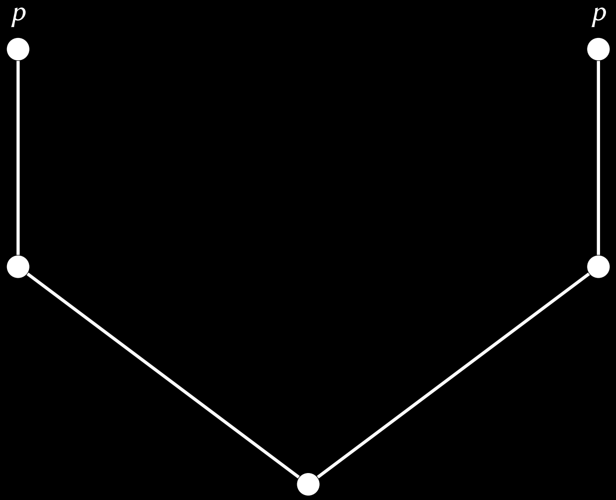
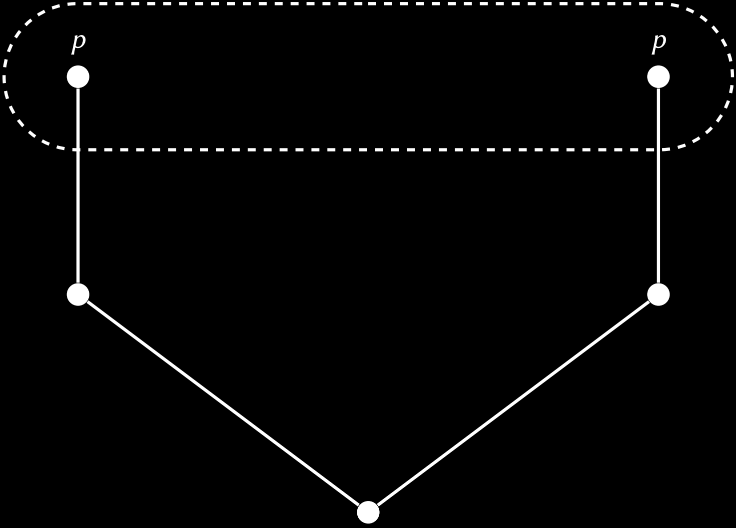


$k \equiv l$ when $v(k) = v(l)$ and $k \leq u$ iff $l \leq u$ for all $u \neq k, l$


p
p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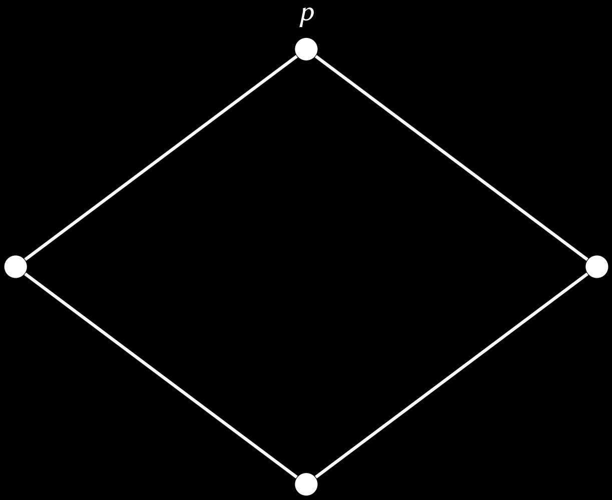
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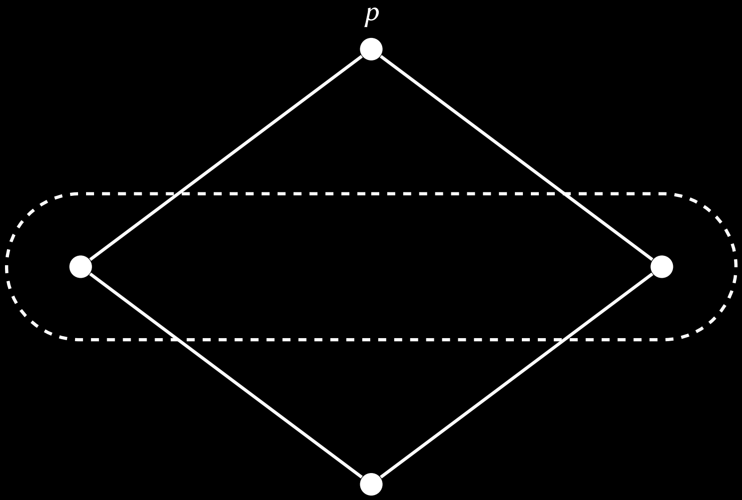
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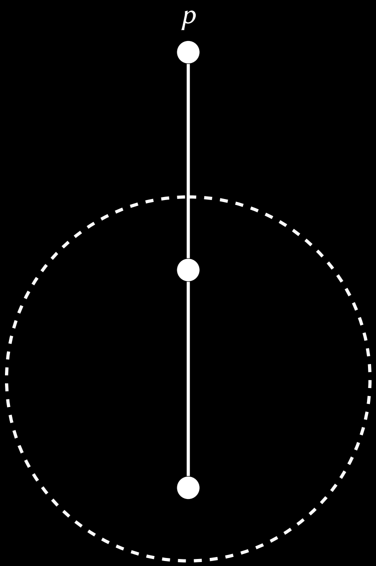
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p

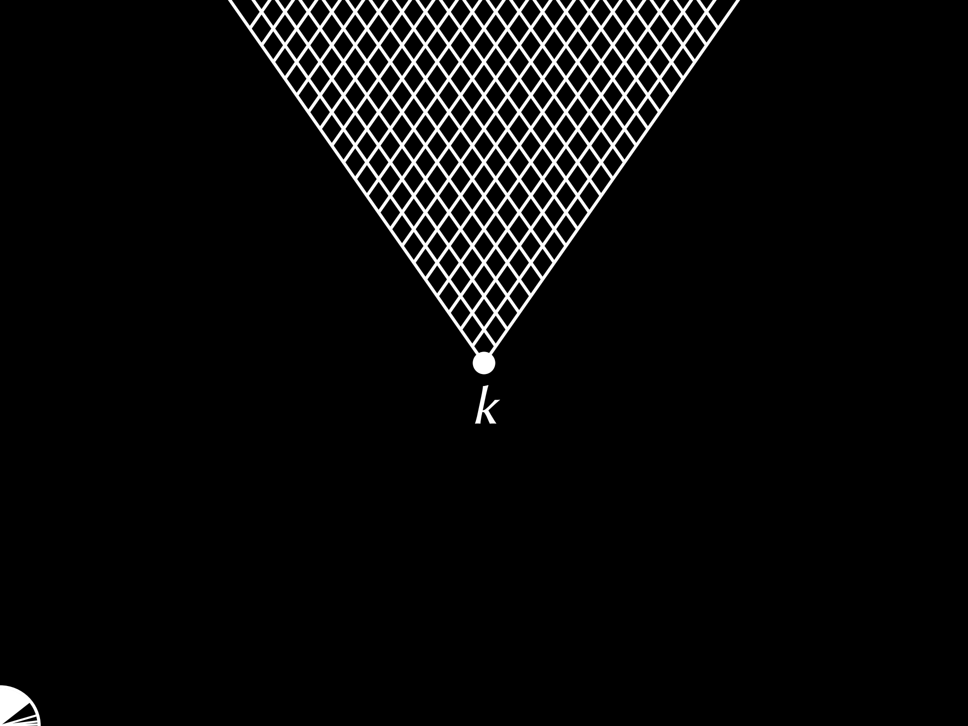
Jankov–de Jongh formulae

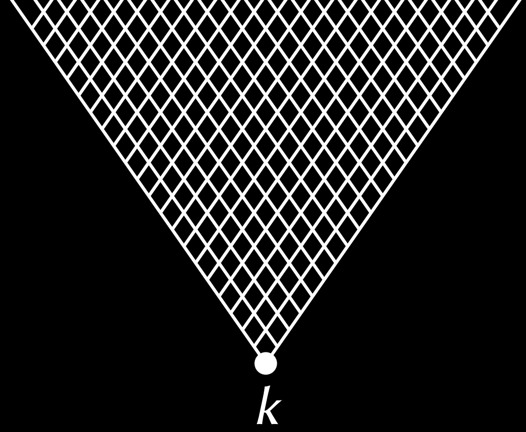
In suitable models have

$$l \Vdash \mathrm{up}\, k \quad \text{iff} \quad k \leq l$$

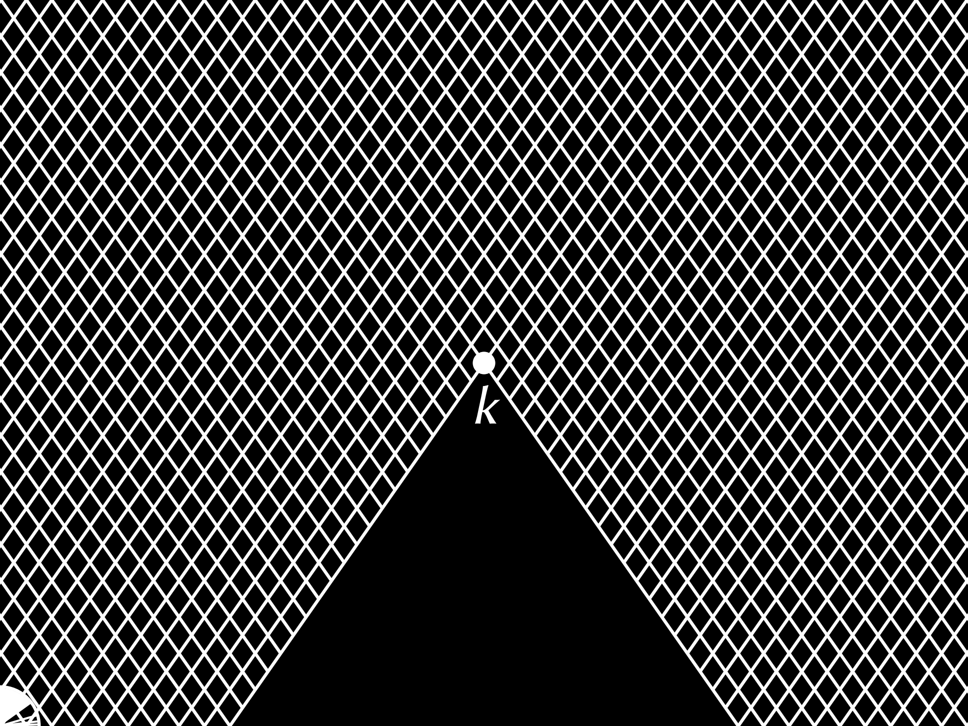
$$l \Vdash \mathrm{nd}\, k \quad \text{iff} \quad l \not\leq k$$

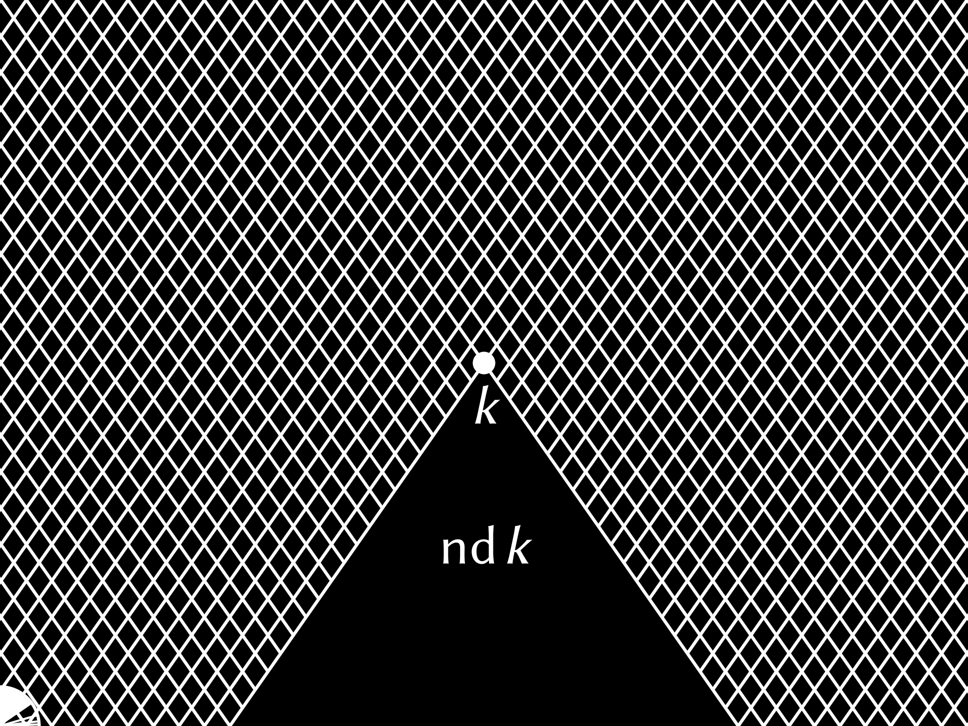

k


k

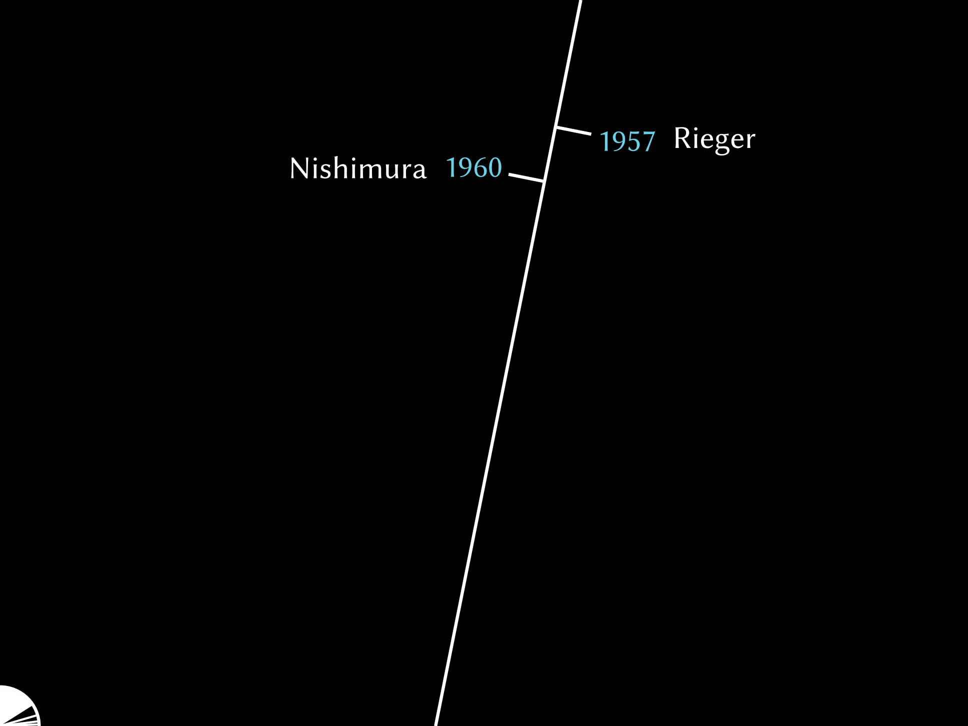


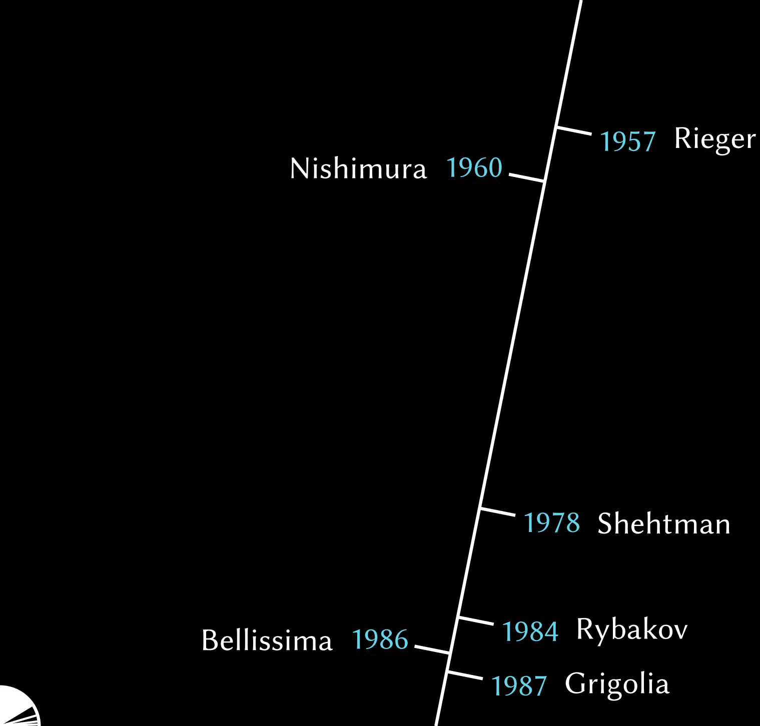
up k


k

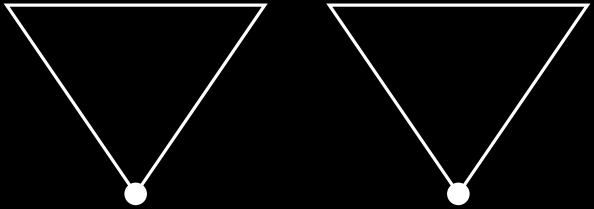

k
nd k

There exists a suitable model containing all rooted finite models.

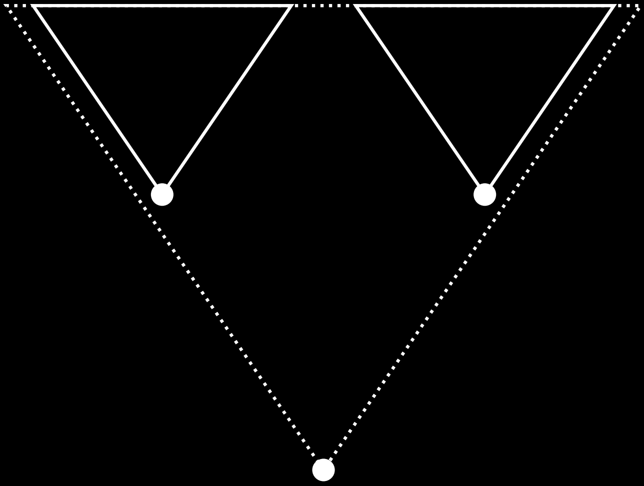

1957 Rieger
Nishimura 1960


1957 Rieger
Nishimura 1960
1978 Shehtman
1984 Rybakov
Bellissima 1986
1987 Grigolia

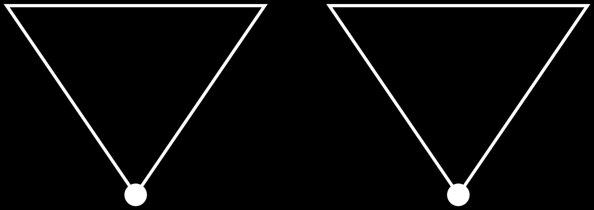
Disjunction Property



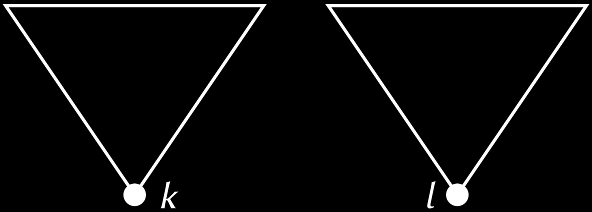
Disjunction Property



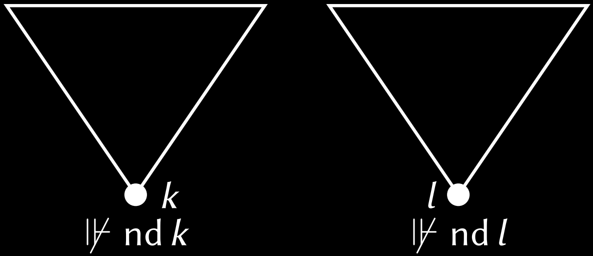
Disjunction Property



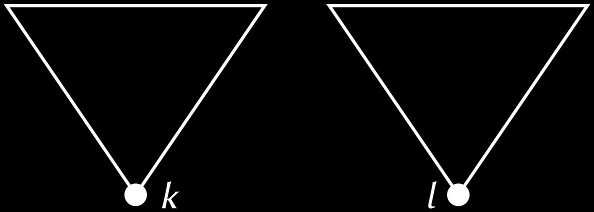
Disjunction Property



Disjunction Property

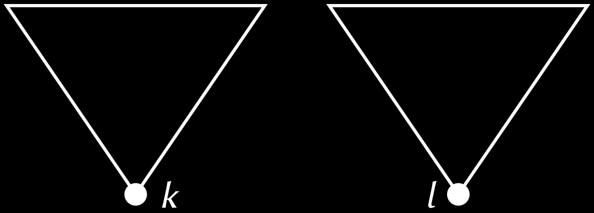


Disjunction Property



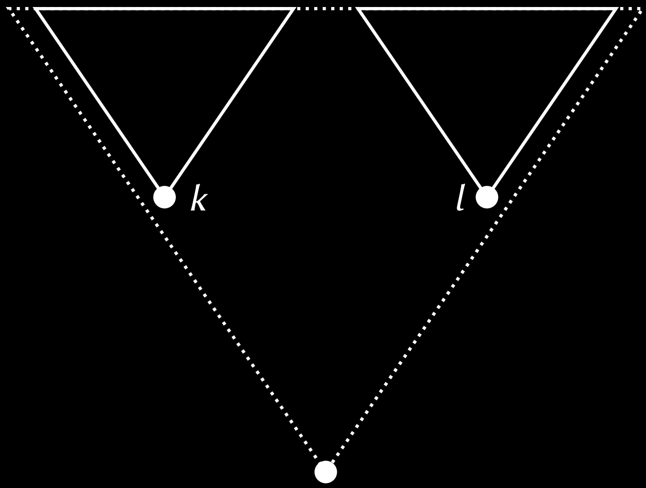
$\nvdash \mathrm{nd}\, k \vee \mathrm{nd}\, l$

Disjunction Property



$\nVdash \mathrm{nd}\, k \vee \mathrm{nd}\, l$

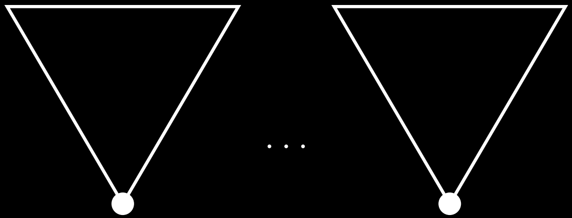
Disjunction Property



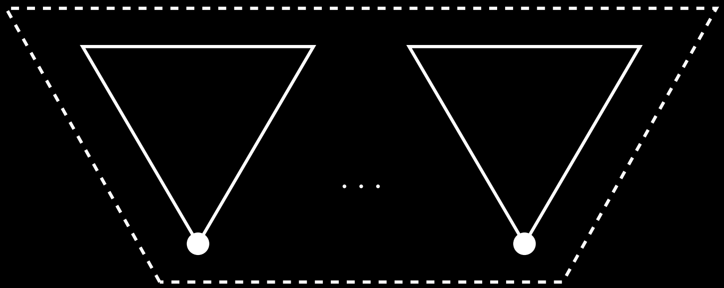
$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\left\{ (\bigvee \Delta \to A) \to C \;\middle|\; C \in \Delta \right\}}$$

Extension Property

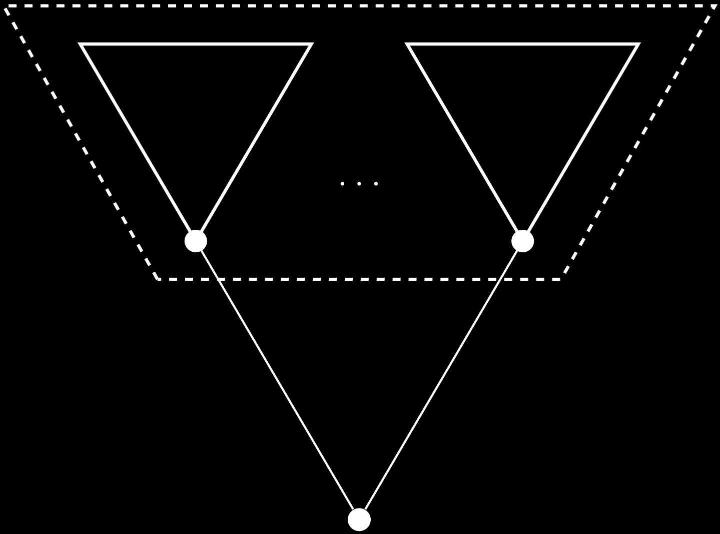
Extension Property



Extension Property



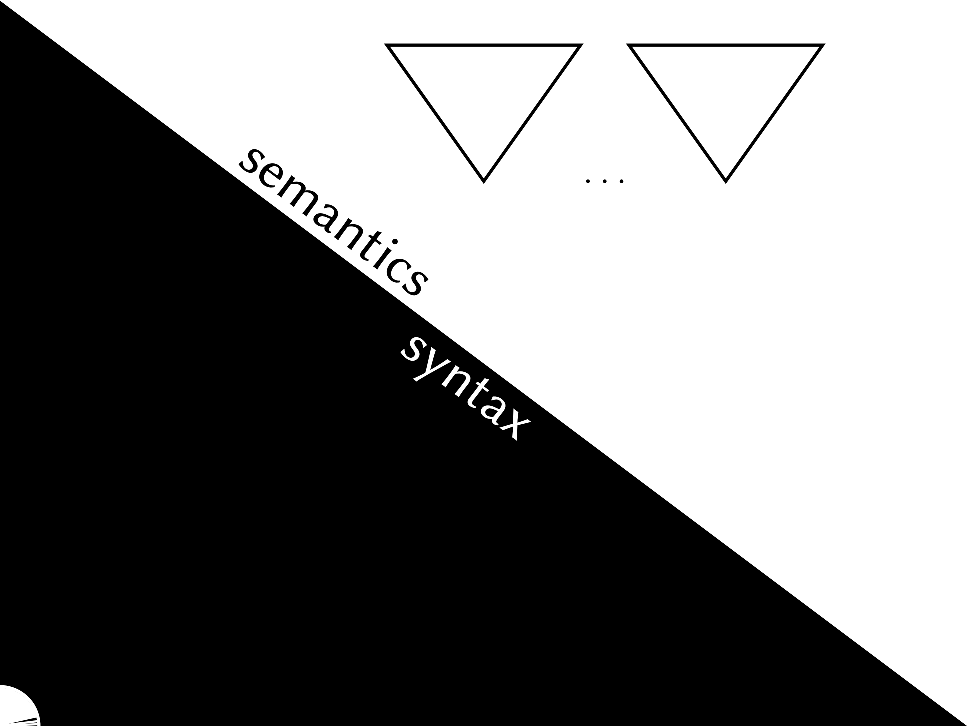
Extension Property



semantics

semantics

syntax


semantics
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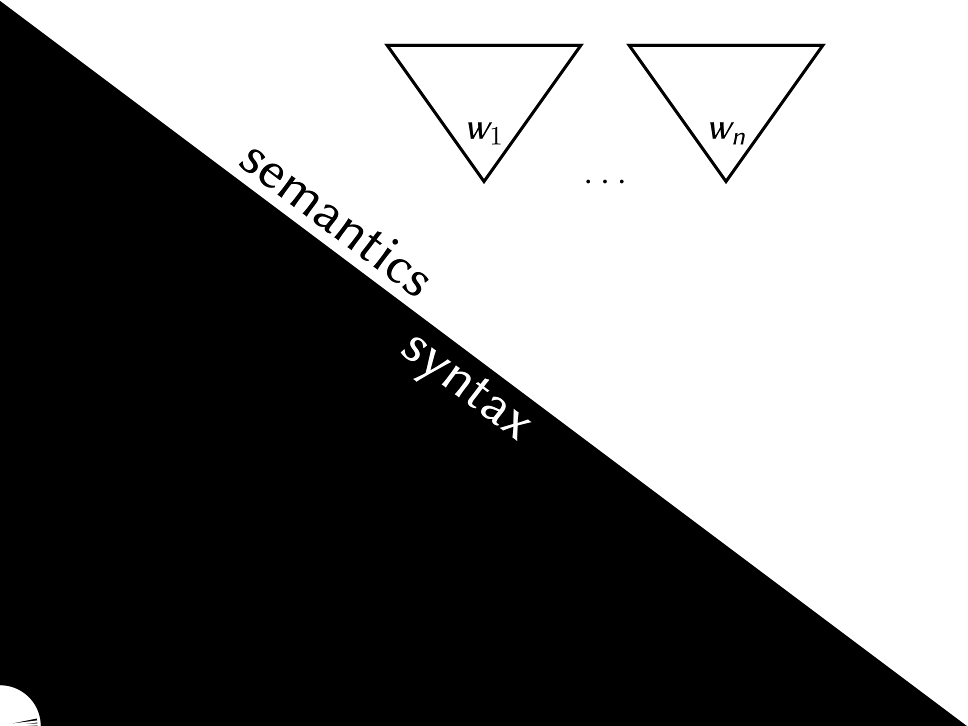
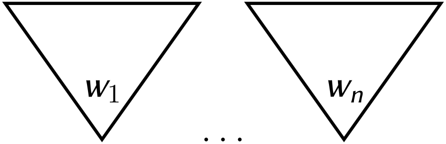
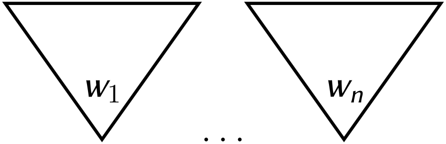

w_1
$\ldots$
w_n
semantics
syntax



semantics

syntax

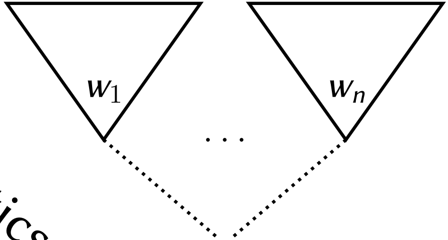
$$\left\{ \left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee\nolimits_{i=1}^{n} \mathsf{up}\, w_i \right) \to \mathsf{nd}\, w_j \right\}_{j=1}^{n}$$



semantics

syntax

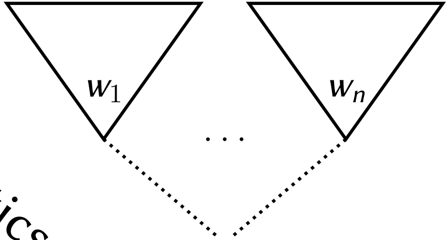
$$\dfrac{\left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \bigvee_{i=1}^{n} \mathsf{nd}\, w_i}{\left\{ \left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \mathsf{nd}\, w_j \right\}_{j=1}^{n}}$$



semantics

syntax

$$\frac{\left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \bigvee_{i=1}^{n} \mathsf{nd}\, w_i}{\left\{ \left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \mathsf{nd}\, w_j \right\}_{j=1}^{n}}$$



semantics

syntax

$$\frac{\left(\bigvee \Delta \to A\right) \to \bigvee \Delta}{\left\{ \left(\bigvee \Delta \to A\right) \to C \right\}_{C \in \Delta}}$$

A is projective when

$\vdash \sigma A$ and $A \vdash \sigma B \equiv B$

for some σ.

A is admissibly saturated when
$A \mathrel{|\!\sim} \Delta$ implies $A \vdash C$
for some $C \in \Delta$.

Ghilardi (1999) and Ghilardi (2004)

A formula is IPC-projective
precisely if it has
the extension property.

A formula B is IPC-projective iff

$$B \vdash (\bigvee \Delta \to A) \to \bigvee \Delta \text{ entails}$$

$$B \vdash (\bigvee \Delta \to A) \to C$$

for some $C \in \Delta$.

Similar characterisations exists for BD_2, T_n and $\mathsf{BD}_2 + \mathsf{T}_n$.

References I

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